Rescaling strange-cluster stars and its implications on gravitational-wave echoes

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Solid states of strange-cluster matter called strangeon matter can form strangeon stars that are highly compact. We show that strangeon matter and strangeon stars can be recast into dimensionless forms by a simple reparametrization and rescaling, through which we manage to maximally reduce the number of degrees of freedom. With this dimensionless scheme, we find that strangeon stars are generally compact enough to feature a photon sphere that is essential to foster gravitational-wave (GW) echoes. Rescaling the dimension back, we illustrate its implications on the expanded dimensional parameter space, and calculate the characteristic GW echo frequencies associated with strangeon stars, showing that the minimum echo frequency is ~8 kHz for empirical parameter space that satisfies the GW170817 constraint, and can reduce to $\mathcal{O}(100)$ Hertz at the extended limit.

I. INTRODUCTION

Recent gravitational wave (GW) observations from compact binary mergers by LIGO/Virgo collaborations [1–7] have significantly advanced our understanding of black holes and compact stars. These binary merger events have inspired many studies on exotic compact objects (ECOs), which are black hole mimickers that share a similar compactness but lack an event horizon [8–14]. Most interests on probes of ECOs are focused on the distinctive signatures from gravitational wave echoes in the postmerger signals [15–35],[1] in which a wave that falls inside the gravitational potential barrier (near the photon sphere) travels to a reflecting boundary of the ECO before returning to the barrier after some time delay.

We want to explore the possibility of GW echoes in the context of realistic compact stars, given the detected binary neutron star merger events. To generate GW echoes, a star object must feature a photon sphere at $R_P = 3M$, where M is the object's mass. The minimum radius for compact stars should be above the Buchdahl's limit $R_B = 9/4M$ [40]. Therefore, GW echo signals are possible if $R_B < R < R_P$. This compactness criterion excludes the realistic neutron

stars [41,42]. To achieve ultracompact stellar structure, people commonly assumed *ad hoc* exotic equations of state (EOS) [42–46] or modified gravity [47–49].

Quark matter, a state comprised of deconfined free-flowing quarks, can possibly exist inside the neutron star core (i.e., hybrid stars [50,51]) or the crust (i.e., inverted hybrid stars [52]), or constitutes entire star called quark star. Strange quark stars [53–58] composed of strange quark matter [59–62] and up-down quark stars [63–72] composed of up-down quark matter [73] can be more compact than neutron stars. As Ref. [74] has shown, physically motivated quark stars can feature GW echoes but require perturbative QCD corrections [75,76] and color superconductivity [77–82] effects to be compact enough. It is interesting to explore whether we can have more compact objects from other physical grounds.

Strangeon matter is similar to strange quark matter where both are composed of a nearly equal number of u, d, s quarks [83–86]. However, strangeon matter has quarks localized as clusters, in a state more like a solid. Strangeon stars [83–94] composed of strangeon matter have intrinsic stiff EOSs that resulted from the repulsive interaction between strangeons at high densities and the nonrelativistic nature of strangeons. The stiff EOS yields large compactness for strangeon stars. They had already been proposed to support massive pulsars ($> 2M_\odot$ [86]) before the announcement of the first massive pulsar PSR J1614-2230 [95]. It is then natural to explore whether strangeon stars can feature GW echoes.

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[1]For probes of ECOs using other methods, see Refs. [36–39].

As for the organization of this paper, we first work out a general dimensionless rescaling for the Lennard-Jones model of strangeon matter. This greatly reduced the number of model parameters from three to one, enabling us to perform a simple but general analysis over the whole parameter space. Then we apply the rescaling scheme to the study of strangeon stars and GW echoes.

II. DIMENSIONLESS RESCALING OF STRANGEON

Following previous studies [83–93], we assume the interaction potential between two strangeons is described by the Lennard-Jones potential [96]:

$$U(r) = 4\epsilon \left[\left(\frac{\sigma}{r} \right)^{12} - \left(\frac{\sigma}{r} \right)^{6} \right], \quad (1)$$

where r is the distance between two strangeons, and σ is the distance when $U(r) = 0$. The parameter ϵ describes the depth of the interaction potential between strangeons. A larger ϵ will then indicate a larger repulsive force at short range and thus maps to a stiffer EOS.

The mass density ρ and pressure density p of zero-temperature dense matter composed of strangeons derived from Lennard-Jones potential [86] reads

$$\rho = 2\epsilon(A_{12}\sigma^{12}n^5 - A_6\sigma^6 n^3) + nN_q m_q, \quad (2)$$

$$p = n^2 \frac{d(\rho/n)}{dn} = 4\epsilon(2A_{12}\sigma^{12}n^5 - A_6\sigma^6 n^3), \quad (3)$$

where $A_{12} = 6.2$, $A_6 = 8.4$, and n is the number density of strangeons. $N_q m_q$ is the mass of a strangeon with N_q being the number of quarks in a strangeon and m_q being the average constituent quark mass. The contributions from degenerate electrons and vibrations of the lattice are neglected due to their expected smallness.

At the surface of strangeon stars, the pressure becomes zero, and we obtain the surface number density of strangeons as $[A_6/(2A_{12}\sigma^6)]^{1/2}$. For convenience, it is transformed into baryon number density, i.e.,

$$n_s = \left(\frac{A_6}{2A_{12}} \right)^{1/2} \frac{N_q}{3\sigma^3}, \quad (4)$$

so that the EOS can be rewritten into a form that depends on parameter set (ϵ, n_s, N_q):

$$\rho = \frac{1}{9}\epsilon \frac{A_6^2}{A_{12}} \left(\frac{N_q^4}{18n_s^4} n^5 - \frac{N_q^2}{n_s^2} n^3 \right) + m_q N_q n, \quad (5)$$

$$p = \frac{2}{9}\epsilon \frac{A_6^2}{A_{12}} \left(\frac{N_q^4}{9n_s^4} n^5 - \frac{N_q^2}{n_s^2} n^3 \right). \quad (6)$$

We find that one can further remove the parameters n_s and N_q by doing the following dimensionless rescaling:

$$\bar{\rho} = \frac{\rho}{m_q n_s}, \quad \bar{p} = \frac{p}{m_q n_s}, \quad \bar{n} = \frac{N_q n}{n_s}, \quad \bar{\epsilon} = \frac{\epsilon}{N_q m_q}, \quad (7)$$

so that

$$\bar{\rho} = \frac{a}{9}\bar{\epsilon} \left(\frac{1}{18}\bar{n}^5 - \bar{n}^3 \right) + \bar{n}, \quad (8)$$

$$\bar{p} = \frac{2a}{9}\bar{\epsilon} \left(\frac{1}{9}\bar{n}^5 - \bar{n}^3 \right), \quad (9)$$

where $a = A_6^2/A_{12} = 8.4^2/6.2 \approx 11.38$. We thus managed to reduce the parameter degree of freedom from 3 (n_s, ϵ, N_q) to simply 1 ($\bar{\epsilon}$). Besides, we note that the rescaled number density at zero pressure always remain $\bar{n} = 3$. Requiring $\bar{\rho}$ to be positive at zero pressure set a theoretical upper bound for $\bar{\epsilon}$: $\bar{\epsilon}_{max}^{theo} = 2/a \approx 0.1757$. However, the value of this upper bound is slightly beyond the empirical expectation $\bar{\epsilon}_{max} \sim 120 \text{ MeV}/(3 \times 310 \text{ MeV}) \approx 0.13$. In the following we will adopt the empirical upper bound $\bar{\epsilon}_{max}^{em} = 0.13$ on this physical ground, with additional comments about results from $\bar{\epsilon}_{max}^{theo}$ at proper places.

III. RESCALING STRANGEON STARS

Inspecting the Tolman-Oppenheimer-Volkoff (TOV) equation [97,98],

$$\frac{dm}{dr} = 4\pi\rho r^2,$$
$$\frac{dp}{dr} = (\rho + p)\frac{m + 4\pi p r^3}{2mr - r^2}, \quad (10)$$

we note that the mass and radius can also be rescaled into dimensionless forms in geometric units ($G = c = 1$)[2]:

$$\bar{m} = m\sqrt{m_q n_s}, \qquad \bar{r} = r\sqrt{m_q n_s}, \quad (11)$$

so that the TOV equation can be converted into the dimensionless form (simply replace nonbarred symbols with barred ones). Solving the dimensionless TOV equation, we obtain the results for the rescaled $\bar{M} - \bar{R}$ relation shown in Fig. 1. One can easily recast it into dimensional form by reversing the rescaling relation Eq. (11). At $\bar{\epsilon} = \bar{\epsilon}_{max}^{em} = 0.13$, we have $(\bar{M}_{TOV}, \bar{R}_{TOV}) \approx (0.149, 0.348)$. Lifting $\bar{\epsilon}$ to 0.175 that is close to $\bar{\epsilon}_{max}^{theo}$, we obtain $(\bar{M}_{TOV}, \bar{R}_{TOV}) \approx (1.28, 2.89)$ correspondingly.

The detailed dependences of maximum compactness M_{TOV}/R_{TOV} and the maximum (rescaled) mass $\bar{M}_{TOV}$ on

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[2]Note that $m_q n_s$, which is in units MeV/fm^3 in natural units, is in the dimension of $[L^{-2}]$ in geometric units here.
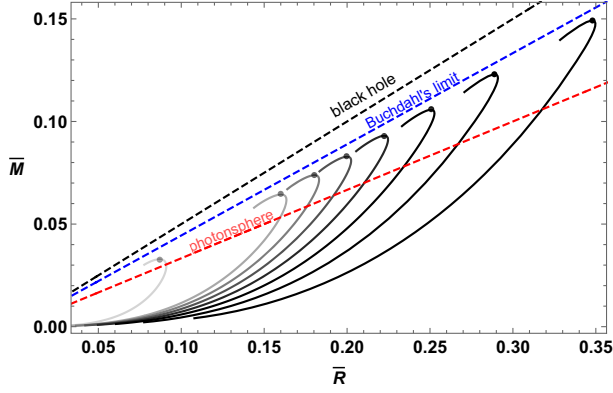
FIG. 1. $\bar{M} - \bar{R}$ of strangeon stars for various $\bar{\epsilon}$, sampling 0.0001–0.13 in equal $\Delta\bar{\epsilon}$ spacing from the lighter black line to the darker black lines, respectively. Solid dots denote the maximum mass configurations.

the single parameter $\bar{\epsilon}$ are illustrated more explicitly in Fig. 2.[3] We find that all $\bar{M} - \bar{R}$ configurations are compact enough to feature a photon sphere while not exceeding the Buchdahl's limit for a large range of $\bar{\epsilon}$ variations.[4] Besides, we see clear positive correlations. As $\bar{\epsilon}$ increases to $\bar{\epsilon}_{\max}^{\text{theo}}$, the maximum mass rapidly grows, while the maximum compactness reaches the Buchdahl's limit.

The rescaled results on tidal deformabilites are shown in Fig. 3. We see that, at a given mass, a larger $\bar{\epsilon}$ increases tidal deformability due to a larger radius, as can be observed from Fig. 1. Besides, we also see that a larger $\bar{\epsilon}$ yields a smaller tidal deformability at the corresponding maximum mass point due to the associated larger compactness.

IV. GW ECHOES FROM STRANGEON STARS

The effective potential of axial gravitational perturbations $\Psi_{s,l}$ in curved background has the general form [21,46]:

$$V(r) = B(r)\left\{\frac{l(l+1)}{r^2} + \frac{1-s^2}{2rA(r)}\left(\frac{B'(r)}{B(r)} - \frac{A'(r)}{A(r)}\right)\right.$$
$$\left. + 8\pi(p(r) - \rho(r))\delta_{s,2}\right\}, \qquad (12)$$

where the azimuthal quantum number $l \geq s$ with $s = 0, \pm1, \pm2$ for scalar, vector, and tensor modes, respectively, and

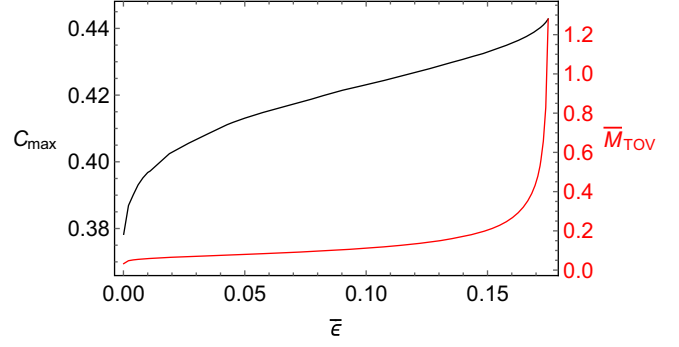$$B(r) = e^{2\Phi(r)}, \qquad A(r) = \frac{1}{1 - 2m(r)/r} \qquad (13)$$

FIG. 2. Maximum compactness $C_{\max} = M_{\text{TOV}}/R_{\text{TOV}}$ (left axis, black) and $\bar{M}_{\text{TOV}}$ (right axis, red) of strangeon stars as a function of $\bar{\epsilon}$.
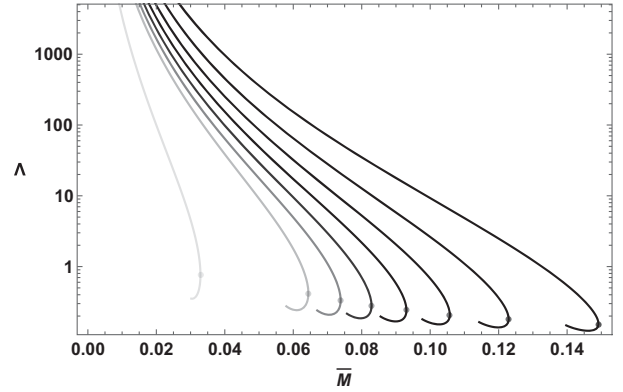

FIG. 3. $\Lambda - \bar{M}$ of strangeon stars for various $\bar{\epsilon}$. The line-color convention follows that of Fig. 1.

as the metric factors of curved line element describing spherical symmetric spacetime: $ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$. $\Phi(r)$ is solved via

$$\frac{d\Phi}{dr} = -\frac{1}{\rho + p}\frac{dp}{dr}, \qquad (14)$$

together with the TOV equation (10). Apparently, we can apply the rescaling relation Eqs. (11) and (7), and perform a further rescaling $\bar{V} = V/(m_q n_s)$ to convert the whole program into a dimensionless form as other barred quantities. We obtain the effective potential of the lowest axial gravitational perturbation mode ($l = s = 2$) in the strangeon star background shown in Fig. 4,[5] which abruptly changes at the star surface, diverges towards star center with an outside peak around $3\bar{M}$, forming a trapping cavity for gravitational waves. We see clearly the trend how the

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[3]Note that here we have extended $\bar{\epsilon}$ to $\bar{\epsilon}_{\max}^{\text{theo}}$ to have a general view.

[4]We examined $\bar{\epsilon}$ as low as 10^{-8} order.

[5]Note that we normalized $\bar{V}(\bar{r})$ with respect to $\bar{V}(\bar{r} = 3\bar{M})$, and $\bar{r}$ with respect to $\bar{M}$, where the rescaling factors cancel and thus would yield same result for the dimensional version.
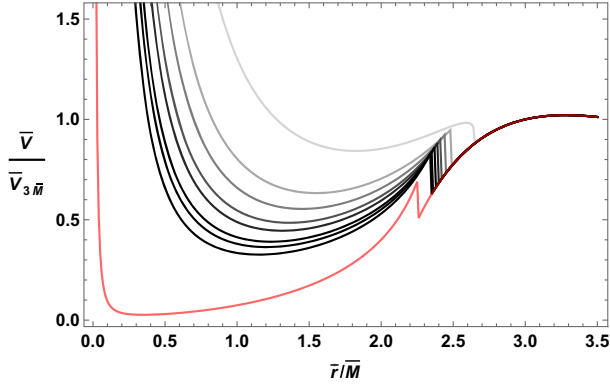
FIG. 4. Radial profiles of effective potentials for axial gravitational perturbations of the $l = s = 2$ mode in strangeon-star background at $M_{\rm TOV}$ points for various $\bar{\epsilon}$. The color convention of black lines follows that of Fig. 1. The red line denotes the $\bar{\epsilon} = 0.175 \approx \bar{\epsilon}_{\rm max}^{\rm theo}$ limit.
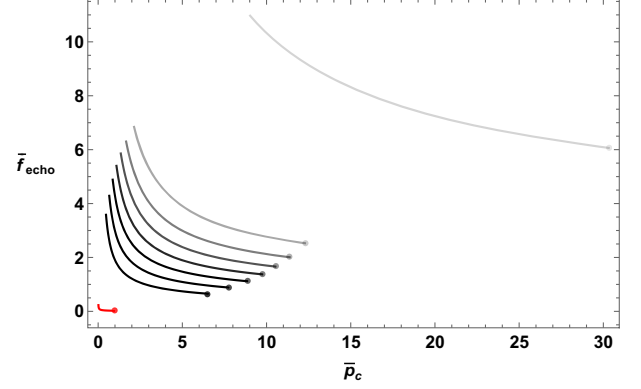

FIG. 5. Rescaled echo frequencies $\bar{f}_{\rm echo}$ as functions of center pressure $\bar{p}_c$ of strangeon stars for various $\bar{\epsilon}$. The color convention of black lines follows that of Fig. 1. The red line denotes the $\bar{\epsilon} = 0.175 \approx \bar{\epsilon}_{\rm max}^{\rm theo}$ limit.

trapping cavity develops and evolves as the parameter $\bar{\epsilon}$ increases.

The characteristic echo time is the light time from the star center to the photon sphere [16–18],

$$\tau_{\rm echo} = \int_0^{3M} \frac{dr}{\sqrt{e^{2\Phi(r)}\left(1 - \frac{2m(r)}{r}\right)}}. \tag{15}$$

We can also do the dimensionless rescaling

$$\bar{\tau}_{\rm echo} = \tau_{\rm echo}\sqrt{m_q n_s}, \tag{16}$$

such that Eq. (15) can also be calculated in a dimensionless approach. After obtaining the echo time, we directly get the GW echo frequency from the relation [16–18]

$$f_{\rm echo} = \frac{\pi}{\tau_{\rm echo}}, \tag{17}$$

and similarly, we can rescale it into the dimensionless form $\bar{f}_{\rm echo}$ via the relation

$$\bar{f}_{\rm echo} = \frac{f_{\rm echo}}{\sqrt{m_q n_s}}. \tag{18}$$

In Fig. 5, we show the results of rescaled GW echo frequencies $\bar{f}_{\rm echo}$ versus the rescaled center pressure $\bar{p}_c$ for the stellar configurations of Fig. 1 that can generate echoes. Note that each curve's left and right ends are truncated at the point where $\bar{R} = 3\bar{M}$, and at the point of maximum mass, respectively.

As $\bar{\epsilon}$ and the central pressure $\bar{p}_c$ increases, $\bar{f}_{\rm echo}$ decreases (due to the increasing compactness) with a lower bound $\bar{f}_{\rm echo}^{\rm min}(\bar{\epsilon})$ set at the $\bar{p}_c$ of the maximum mass point. At

$\bar{\epsilon} = \bar{\epsilon}_{\rm max}^{\rm em} = 0.13$, we have the minimum echo frequency $\bar{f}_{\rm echo}^{\rm min} = 0.655$, translating to

$$f_{\rm echo}^{\rm min} \approx 2\left(\frac{n_s}{0.24 \ {\rm fm}^{-3}}\right)^{1/2} {\rm kHz} \quad {\rm for} \ \bar{\epsilon} = \bar{\epsilon}_{\rm max}^{\rm em}. \tag{19}$$

Lifting $\bar{\epsilon}$ to 0.175, which is close to $\bar{\epsilon}_{\rm max}^{\rm theo}$, we obtain $\bar{f}_{\rm echo}^{\rm min} = 0.023$, mapping this to

$$f_{\rm echo}^{\rm min} \approx 67\left(\frac{n_s}{0.24 \ {\rm fm}^{-3}}\right)^{1/2} {\rm Hz} \quad {\rm for} \ \bar{\epsilon} \approx \bar{\epsilon}_{\rm max}^{\rm theo}. \tag{20}$$

Interestingly, we see that, in this extreme limit, the minimum echo frequencies lie well within the sensitivity range of LIGO [1,46].

V. DIMENSIONAL PARAMETER SPACE

In Fig. 6, we show the derived quantities in dimensional forms ($f_{\rm echo}$, $M_{\rm TOV}$) and (Λ, C) in dimensional parameter space of (ϵ, n_s) by rescaling back previous simple dimensionless results using relations Eqs. (7) and (11).

Figure 6 manifests the following apparent scaling behavior:

(1) Decreasing ϵ for given N_q or increasing N_q for given ϵ is equivalent in terms of $\bar{\epsilon}$.

(2) Dimensionless quantities like the compactness $C = M/R$ should be independent of n_s. This explains why the blue dotted lines are flat.

(3) Dimensional quantities follows the scaling relation dictated by Eqs. (7) and (11). For example, the maximum mass $M_{\rm TOV}$ (and corresponding radius $R_{\rm TOV}$) scale as $\sqrt{n_s}$.

From Fig. 6, we see explicitly that the minimum echo frequency is ∼8 kHz for the parameter space of large ϵ and small n_s, which satisfies the GW170817 tidal
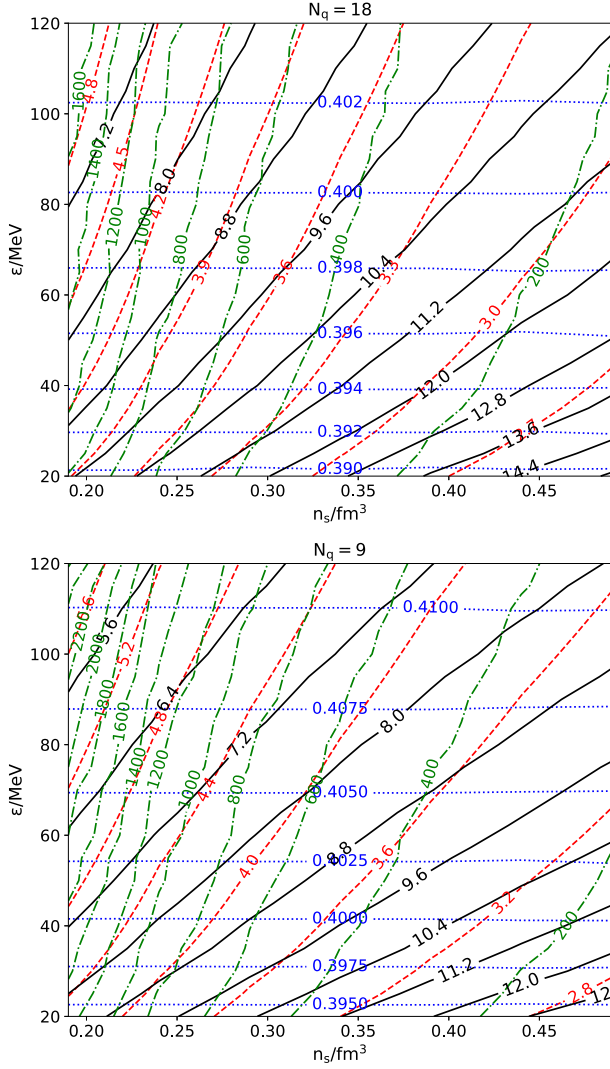
FIG. 6. Physical parameter space for (top) $N_q = 18$ and (bottom) $N_q = 9$. Black lines denote f_{echo}/kHz, with red lines denoting the maximum masses $M_{TOV}/M_\odot$, green lines for the tidal deformabilities Λ at $1.4M_\odot$ and blue dotted lines for the maximal compactness M_{TOV}/R_{TOV}.

deformability constraint $\Lambda(1.4M_\odot) \lesssim 800$ and can be reduced to 5 kHz if the GW170817 constraint is dropped, i.e., if assuming the detected stars in the binary merger are not strangeon stars.

VI. SUMMARY

We worked out a first rescaling scheme that enables us to maximally reduce the number of free parameters into a single parameter $\bar{\epsilon}$ for strangeon matter. Utilizing this scheme, we demonstrated that strangeon stars composed of strangeon matter generally have very large compactness with large $\bar{\epsilon}$ in most of its parameter space. We showed that all strangeon stars can meet the compactness condition for generating GW echoes, i.e., they feature a photon sphere within Buchdahl's limit. The minimum echo frequencies are of a few kilohertz for the empirical range of $\bar{\epsilon}$, and can reduce to $O(100)$ Hertz if $\bar{\epsilon}$ is extended to its allowed limit. We explicitly constructed the corresponding dimensional parameter space of ϵ and n_s with variations of N_q in their empirical range and demonstrated that $f_{echo}^{min} \approx 8$ kHz for realistic parameter spaces, which can satisfy astrophysical constraints like GW170817 and can reduce to 5 kHz if the latter constraint is dropped.

It is generally expected that including the star-rotation effect can slightly reduce the echo frequencies [42,46]. For strangeon stars, we expect rotation would yield a similar reduction of f_{echo}, potentially reducing frequencies to what LIGO can detect, considering our obtained $f_{echo}^{min} \approx 5 \sim 8$ kHz for the realistic nonrotating case is not very far from its detection limit. Besides, strangeon stars may have hybrid structures, likely a strangeon matter crust and a strange quark matter core resulting from a first-order phase transition at high densities. Such a new type of hybrid stars may be also compact enough to signal GW echoes. We leave these interesting possibilities for future studies.

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