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H-cluster stars

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ABSTRACT

The study of dense matter at ultrahigh density has a very long history, which is meaningful for us to understand not only cosmic events in extreme circumstances but also fundamental laws of physics. It is well known that the state of cold matter at supranuclear density depends on the non-perturbative nature of quantum chromodynamics (QCD) and is essential for modelling pulsars. A so-called H-cluster matter is proposed in this paper as the nature of dense matter in reality.

In compact stars at only a few nuclear densities but low temperature, quarks could be interacting strongly with each other there. That might render quarks grouped in clusters, although the hypothetical quark clusters in cold dense matter have not been confirmed due to the lack of both theoretical and experimental evidence. Motivated by recent lattice QCD simulations of the H-dibaryons (with structure *uuddss*), we therefore consider here a possible kind of quark clusters, H-clusters, that could emerge inside compact stars during their initial cooling as the dominant components inside (the degree of freedom could then be H-clusters there). Taking into account the in-medium stiffening effect, we find that at baryon densities of compact stars H-cluster matter could be more stable than nuclear matter. We also find that for the H-cluster matter with lattice structure, the equation of state could be so stiff that it would seem to be ‘superluminal’ in the most dense region. However, the real sound speed for H-cluster matter is in fact difficult to calculate, so at this stage we do not put constraints on our model from the usual requirement of causality.

We study the stars composed of H-clusters, i.e. H-cluster stars, and derive the dependence of their maximum mass on the in-medium stiffening effect, showing that the maximum mass could be well above 2 $M_{\odot}$ as observed and that the resultant mass–radius relation fits the measurement of the rapid burster under reasonable parameters. Besides a general understanding of different manifestations of compact stars, we expect further observational and experimental tests for the H-cluster stars in the future.

Key words: dense matter – elementary particles – equation of state – stars: neutron – pulsars: general.

1 INTRODUCTION

Possible matter at the highest density, *limited by the sizes of electrons and nuclei*, was incidentally speculated in a seminal paper (Fowler 1926) about a decade after Rutherford suggested his model of the atom. Compact objects, especially at density as high as nuclear matter density, are gradually focused on by astronomers and physicists, the study of which opens a unique window that relates fundamental particle physics and astrophysics. It is worth noting that above the saturated nuclear matter density, ρ_0, the state of matter is still far from certainty, whereas it is essential for us to explore the nature of pulsars. Historically, at average density higher than $\sim 2\rho_0$, the quark degrees of freedom inside would not be negligible, and such compact stars are then called quark stars (Itoh 1970; Alcock, Farhi & Olinto 1986; Haensel, Zdunik & Schaefer 1986). Bodmer–Witten conjecture says that strange quark matter (composed of *u*, *d* and *s* quarks) could be more stable than nuclear matter (Bodmer 1971; Witten 1984). Although the effect of non-perturbative quantum chromodynamics (QCD) makes it difficult to derive the real state of cold quark matter, the existence of quark stars cannot be ruled out yet, neither theoretically nor observationally (see a review in Weber 2005).

Although quark matter at high density but low temperature is difficult to be created in laboratory as well as difficult to be studied by pure QCD calculations, some efforts have been made to understand the state of cold quark matter and quark stars. The MIT bag model treats the quarks as relativistic and weakly interacting

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particles, which is the most widely used model for quark stars (Alcock et al. 1986; Haensel et al. 1986). The colour superconductivity (CSC) state is currently focused on perturbative QCD as well as QCD-based effective models (Alford 2008). In most of these models, quark stars are usually characterized by soft equations of state, because the asymptotic freedom of QCD tells us that as the energy scale goes higher, the interaction between quarks becomes weaker and weaker.

In cold quark matter at realistic baryon densities of compact stars ($\rho \sim 2$–$10\rho_0$), however, the energy scale is far from the region where the asymptotic freedom approximation could apply. In this case, the interaction energy between quarks could be comparable to the Fermi energy, so that the ground state of realistic quark matter might not be that of Fermi gas (see a discussion given in Xu 2010). Some evidence in heavy ion collision experiments also shows that the interaction between quarks is still very strong even in the case of hot quark–gluon plasma (Shuryak 2009). It is then reasonable to infer that quarks could be coupled strongly also in the interior of those speculated quark stars, which could make quarks to condensate in position space to form quark clusters.[1] The observational tests from polarization, pulsar timing and asteroseismology have been discussed (Xu 2003), and it is found that the idea of clustering quark matter could provide us a way to understand different manifestations of pulsars. The realistic quark stars could then be actually 'quark-cluster stars'.[2] An interesting suggestion is that quark matter could be in a solid state (Xu 2003; Horvath 2005; Owen 2005; Mannarelli, Rajagopal & Sharma 2007), and for quark-cluster stars, solidification could be a natural result if the kinetic energy of quark clusters is much lower than the interaction energy between the clusters.

Quark clusters may be analogized to hadrons, and in fact some authors have discussed the stability of hadron bound states. A dihyperon with quantum numbers of $\Lambda\Lambda$ (H-dibaryon) was predicted to be a stable state or resonance (Jaffe 1977), and an 18-quark cluster (quark-alpha, Q_α) being completely symmetric in spin, colour and flavour spaces was also proposed (Michel 1988). H-dibaryon in lattice QCD simulations, although no direct evidence from experiments, provides us a specific kind of quark clusters that could be very likely to exist inside quark stars. In fact, H-dibaryons have been studied for years as a possible kind of multiquark compound states. The non-relativistic quark-cluster model was introduced to study the binding energy of H-clusters (Straub et al. 1988). The interaction between H-clusters was investigated by employing one-gluon-exchange potential and an effective meson-exchange potential, and a short-range repulsion was found (Sakai et al. 1997). Recently, H-dibaryon, with binding energy of about 10–40 MeV, has been found in lattice QCD simulations by two independent groups (Beane et al. 2010; Inoue et al. 2010), and STAR preliminary results show also possible stable H-dibaryons from Λ–Λ correlations (Huang, private communication).

[1] Besides this top-down scenario, i.e. an approach from the deconfined quark state with the inclusions of stronger and stronger interaction between quarks, one could also start from the hadronic state (a bottom-up scenario): strangeness may play an important role in *gigantic* nuclei so that the degree of freedom would not be nucleon but quark cluster with strangeness.

[2] Strictly speaking, quark-cluster stars are *not* quark stars if one thinks that the latter are composed of free quarks. Nevertheless, in this paper, we temporarily consider quark-cluster stars as a very special kind of quark stars since (1) both kinds of stars are self-bound and quark-cluster stars manifest themselves similar to quark stars rather than gravity-bound neutron stars, and (2) the quark degree of freedom plays a significant role in determining the equation of state and during the formation of quark-cluster stars.

Strange quark matter, with light flavour symmetry, has nearly equal numbers of *u*, *d* and *s* quarks. If quark clusters are the dominant components of strange quark matter, then it is natural to conjecture that each quark cluster could be composed of almost equal numbers of *u*, *d* and *s* quarks. During the initial cooling of a quark star, the interaction between quarks will become stronger and stronger, and then H-clusters (six-quark clusters with the same structure as H-dibaryons *uuddss*) would emerge from the combination of three-quark clusters (with structure *uds* as Λ particles), due to the attraction between them. If the light flavour symmetry is ensured, then the dominant components inside the stars are very likely to be H-clusters. In our previous work about the quark-cluster stars, the number of quarks inside each quark cluster $N_{\rm q}$ is taken to be a free parameter (Lai & Xu 2009b), and as the further study in this paper, we realistically specify the quark clusters to be H-clusters.

There could be other kind of particles with strangeness, such as kaons and hyperons. Kaon condensate would probably reduce the maximum mass of the stars and hyperons heavier than Λ^0 that would not have large enough number densities, both of which would not have significant effect on the stars' global structure. We neglect them in this paper as the first step towards the structure of this specific kind of quark-cluster stars, and the effect of all possible particles should be taken into account in our further studies.

To study the H-cluster stars, we assume that the interaction between H-clusters is mediated simply by σ and ω mesons and introduce the Yukawa potential to describe the H–H interaction (Faessler et al. 1997), and then derive the mass–radius relations of H-cluster stars in different cases of the in-medium stiffening effect and surface density. Under a wide range of parameter space, the maximum mass of H-cluster stars can be well above $2\,M_\odot$, and therefore such compact stars cannot be ruled out even though some pulsars with mass as high as $2\,M_\odot$ are found (Lai & Xu 2011). Moreover, the observations (e.g. pulsar mass) could help us constrain the H–H interaction in dense matter.

The paper is arranged as follows. The existence and localization of H-clusters inside compact stars are discussed in Section 2. The equation of state and the global structure of H-cluster stars are given in Section 3, including the dependence of their maximum mass on the in-medium stiffening effect and surface density of the star. Some issues about the H-cluster stars are discussed in Section 4, and we make conclusions in Section 5.

2 H-CLUSTERS INSIDE COMPACT STARS

The state of matter of compact stars is essentially a problem of non-perturbative QCD, with energy scale below 0.8 GeV (corresponding to mass density below $10\rho_0$). If the interaction between quarks could be strong enough to group them into clusters, the quark-clustering phase should be very different from the CSC phase under perturbative QCD, and it could also be different from the normal hadron phase if the quark matter has light flavour symmetry. Whether H-clusters could be the dominant component inside compact stars is an interesting but difficult problem, and here we just make some rough estimation about their existence and quantum effect. We find that at densities inside pulsars, the in-medium effect could make H-cluster matter to be more stable than nucleon matter, where H-clusters are stable against decaying to nucleons. Moreover, they could be localized rather than that of Bose–Einstein condensation, and such a localization of quark clusters could lead to a classical crystalline structure to form solid state. In addition, the lattice vibration would not dissolve H-clusters.

2.1 The stability of H-cluster matter

Whether the Bodmer–Witten conjecture is true or not is difficult for us to solve from the first principle. Here we demonstrate that H-cluster matter could be stable with respect to transforming into nucleon matter at the same density, by assuming the Brown–Rho scaling.

In dense matter (ignoring the mass difference between neutrons and protons), the masses of neutrons and mesons satisfy the scaling law $m_{\rm N}^*/m_{\rm N} = m_{\rm M}^*/m_{\rm M}$, where $m_{\rm N}$ and $m_{\rm M}$ denote the mass of nucleons and mesons, and the masses with and without asterisks stand for in-medium values and free-space values, respectively. This is called the Brown–Rho scaling (for a review, see Brown & Rho 2004). We suppose that the Brown–Rho scaling holds for H-dibaryons, which have the same mass scaling as nucleons

$$m_{\rm N}^*/m_{\rm N} = m_{\rm M}^*/m_{\rm M} = m_{\rm H}^*/m_{\rm H} = 1 - \alpha_{\rm BR}\frac{n}{n_0}, \quad (1)$$

where n denotes the number density of H-dibaryons, n_0 denotes the number density of saturated nuclear matter and $\alpha_{\rm BR}$ is the coefficient of scaling. For nuclear matter, $\alpha_{\rm BR}$ is found to be 0.15 for mesons and 0.2 for nucleons by fitting experimental data (Brown, Sethi & Hintz 1991). The density dependence of $\alpha_{\rm BR}$ is still unknown, especially at supranuclear density. In our following calculations, we treat $\alpha_{\rm BR}$ as a parameter in the range between 0.1 and 0.2, and for simplicity we assume that its value is the same for both mesons and H-dibaryons.

At first, for simplicity we do not consider the interaction of nucleons when comparing the energy per baryon of nuclear matter and H-cluster matter. For the system composed of nucleons in weak equilibrium at densities higher than ρ_0, the dominant component is neutrons. Taking into account the in-medium effect, the energy per baryon of neutron matter is $E/A = \sqrt{p_{\rm n}^2 + m_{\rm n}^{*2}} + E_{\rm sym}$, where $n_{\rm n}$ is number density of neutrons, $p_{\rm n} = (3\pi^2 n_{\rm n})^{1/3}$ is the Fermi momentum of neutrons, $E_{\rm sym}$ is the symmetry energy per baryon, and we use the expression $E_{\rm sym} = 31.6(n_{\rm n}/n_0)^{1.05}$ MeV (Chen, Ko & Li 2005). We also consider the nucleon matter with equal number density of neutrons, protons and electrons, $n_{\rm n} = n_{\rm p} = n_{\rm e}$, where the symmetry energy is vanishing but the electron Fermi energy $p_{\rm e} = (3\pi^2 n_{\rm e})^{1/3}$ is high, and the energy per baryon is $E/A = \sqrt{p_{\rm n}^2 + m_{\rm n}^{*2}} + p_{\rm e}/2$. The energy per baryon of H-cluster matter is $E/A = \epsilon/n_{\rm B}$, where ϵ is the energy density of H-cluster matter calculated in Section 3.1 and $n_{\rm B}$ is the baryon number density ($n_{\rm B} = 2n$). We set the free-space value for the mass of H-cluster to be $m_{\rm H} = 2m_\Lambda - 20\,{\rm MeV} = 2210\,{\rm MeV}$.

Fig. 1 shows the dependence of E/A on baryon number density $n_{\rm B}$, in the cases $\alpha_{\rm BR} = 0.15$, for H-matter, neutron matter and nucleon matter with $n_{\rm n} = n_{\rm p} = n_{\rm e}$. We can see that H-matter is more stable than nuclear matter when the number density is larger than $2n_0$ (due to the in-medium effect, the rest-mass energy density of H-cluster matter at $2n_0$ is about $3.3\rho_0$). If the stability condition is satisfied, H-clusters would not decay to nucleons, where the in-medium effect plays the crucial role in stabilizing H-cluster matter. The decrease of the mass of H-dibaryons with increasing densities could be equivalent to the increase of binding energy of H-dibaryons, which makes H-clusters to be more stable.

Actually, the stability condition we discussed above is applied to the surface of H-cluster stars, where the pressure is vanishing, and the densities we choose above can be seen as the surface densities (given the surface density we can get coefficients in the interaction potential, see Section 3.1). At high densities, the interaction between nucleons would become significant to resist gravity. To compare the stability of H-cluster matter and nuclear matter at high densities inside stars, we choose one of the models which describe the state of nuclear matter (Niu & Gao 2010). We compare the chemical potential per baryon $\mu_{\rm B} = (\epsilon + P)/n_{\rm B}$ of H-cluster matter with that of nuclear matter (Niu & Gao 2010), when pressure is non-zero. The results are shown in Fig. 2, where for H-cluster matter, in three cases $\alpha_{\rm BR} = 0.1$, 0.15 and 0.2, the surface density $n_{\rm s} = 2n_0$, and the densities are chosen in the range where the stars could be gravitationally stable (according to Fig. 5), and for nuclear matter the densities are chosen in the range as the widest range among the three, with $n_{\rm B} \simeq 8.3n_0$. We can see that H-cluster matter would be more energetically favourable than nuclear matter when $\mu_{\rm B} \lesssim 2000\,{\rm MeV}$. Whether there would be phase transition from H-cluster matter to some other forms of matter at higher densities is unknown because of the ignorance of state of matter at densities beyond several times of n_0, so we do not extrapolate the curve of nuclear matter to larger $\mu_{\rm B}$ in Fig. 2 and consider the possible phase transition from this figure. As discussed in Section 4.5, although the

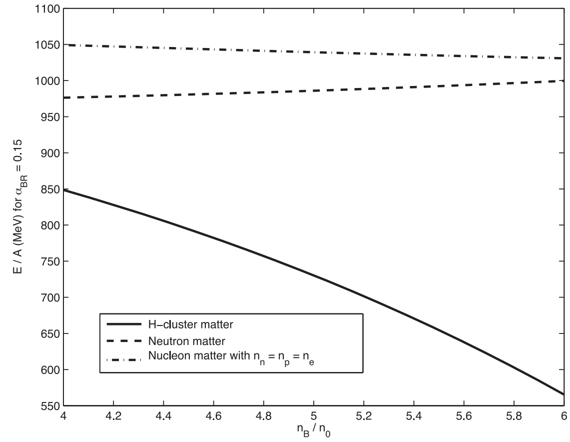

Figure 1. The dependence of energy per baryon E/A on baryon number density $n_{\rm B}$, for H-matter (solid line), neutron matter (dashed line) and nucleon matter with $n_{\rm n} = n_{\rm p} = n_{\rm e}$ (dash–dotted line), in the case $\alpha_{\rm BR} = 0.15$. It is evident that H-particles would be stable when the number density is above $2n_0$.

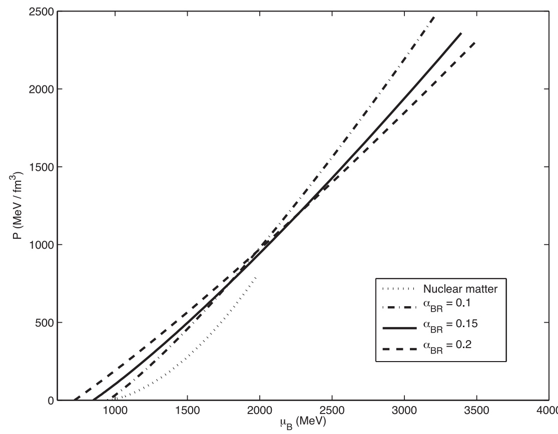

Figure 2. The chemical potential for baryon $\mu_{\rm B}$, for nuclear matter (Niu & Gao 2010) (dotted line), and H-cluster matter with $\alpha_{\rm BR} = 0.1$ (dash–dotted line), $\alpha_{\rm BR} = 0.15$ (solid line) and $\alpha_{\rm BR} = 0.2$ (dashed line), where the surface density $n_{\rm s} = 2n_0$. For H-cluster matter, the densities are chosen in the range where the stars could be gravitationally stable, and for nuclear matter the densities are chosen in the range as the widest range of baryon number densities among the three (with $n_{\rm B} \simeq 8.3n_0$).

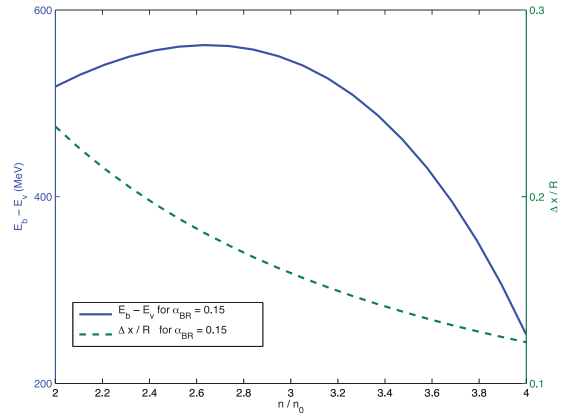

Figure 3. The comparison of binding energy $E_{\rm b}$ and vibration energy $E_{\rm v}$ (solid line), and the ratio of the deviation from the equilibrium position Δx to the distance between two nearby H-clusters R (dashed line), as the function of number density n, in the case $\alpha_{\rm BR} = 0.15$. If $E_{\rm b} - E_{\rm v} > 0$, H-clusters in crystalline structure would be stable against lattice vibration; if $\Delta x/R < 1$, the quantum effect would not be significant and the Bose–Einstein condensate would not take place.

state of dense matter at ultrahigh density is uncertain, three-flavour symmetry may result in a ground state of matter.

2.2 Crystallization of H-cluster matter

Under the interaction, H-clusters could be localized and behave like classical particles. In the core of a neutron star, H-clusters could also appear, and the existence of H-clusters inside neutron stars has been studied in relativistic mean-field theory (Faessler et al. 1997). It was found that when the potential between H-clusters is negative enough, then a substantial number density of H-clusters will reduce the maximum mass of neutron stars if Bose–Einstein condensation happens (Glendenning & Schaffner-Bielich 1998).

However, due to the strong interaction, H-clusters would be localized like classical particles in crystal lattice, and the quantum effect would be negligible. One H-cluster is under the composition of interaction from its neighbouring H-clusters, which forms a potential well. The energy fluctuation makes this H-cluster oscillate about its equilibrium position with the deviation Δx, i.e. the vibration energy $E_{\rm v}$ can be derived as $E_{\rm v} \simeq \hbar^2/(m_{\rm H}\Delta x^2) \simeq k\Delta x^2$, where $k \simeq \partial^2 V(r)/\partial r^2$, and r is the distance of two neighbouring H-clusters. We use the H–H interaction in equation (2), and estimate $E_{\rm v}$ and Δx at different densities. Taking $\alpha_{\rm BR} = 0.15$ as an example, the results are shown in Fig. 3. The density range we choose here is narrow, since we will see in Section 3.2 that the density of a stable star will be well below $4\rho_0$ in the case $\alpha_{\rm BR} = 0.15$. In the proper density range, the vibration energy is smaller than the binding energy of H-clusters, which means that the fluctuations about the lattice would not dissolve H-clusters. On the other hand, the distance between two nearby H-clusters $R = n^{-1/3}$ (with n the number density of H-clusters) is larger than Δx, which means that the quantum effect could not be significant and the Bose–Einstein condensate would not take place.

H-clusters are localized because each of them feels an ultrastrong repulsion in every direction around it, and such localization could lead to a crystalline structure. In fact, the relation between hard-core potential and crystallization was discussed previously (e.g. Canuto 1975). The almost infinitely strong repulsion is certainly an ideal case, but in real world the short range of H–H interaction could be still strong enough to localize them.

3 THE GLOBAL STRUCTURE OF H-CLUSTER STARS

We propose a possible kind of quark-cluster stars totally composed of H-clusters, i.e. H-cluster stars. H-cluster stars could have different properties from neutron stars and conventional quark stars, such as the radiation properties, cooling behaviour and global structure. In this paper, we only focus on the global structure of H-cluster stars, deriving the mass–radius relation based on the equation of state.

3.1 H–H interaction and equation of state

The interaction between H-clusters has been studied under the Yukawa potential with σ and ω coupling (Faessler et al. 1997), and we adopt this form of interaction here

$$V(r) = \frac{g_{\omega \rm H}^2}{4\pi}\frac{{\rm e}^{-m_\omega r}}{r} - \frac{g_{\sigma \rm H}^2}{4\pi}\frac{{\rm e}^{-m_\sigma r}}{r}, \qquad (2)$$

where $g_{\omega \rm H}$ and $g_{\sigma \rm H}$ are the coupling constants of H-clusters and meson fields. The numerical result of the potential between two H-dibaryons shows a minimum at $r_0 \approx 0.7$ fm with the depth $V_0 \approx -400$ MeV (Sakai et al. 1997), which means that, to get the minimal point, two H-dibaryons should be very close to each other.

Nevertheless, the medium effect in dense matter could change those properties. In dense nuclear matter, the effective meson masses $m_{\rm M}^*$ satisfy the Brown–Rho scaling law of equation (1) (Brown & Rho 2004) This shows the in-medium effect that stiffens the interparticle potential by reducing the meson effective masses, and m_σ and m_ω in equation (2) should be replaced by m_σ^* and m_ω^*. In addition, the mass of H-dibaryons $m_{\rm H}$ also obeys the same scaling law.

Given the potential between two H-clusters, we can get the energy density by taking into account all of the contributions from H-clusters in the system. In the case of a strong repulsive core, each H-cluster could be trapped inside the potential well as demonstrated before. Assuming that the localized H-clusters form the simple cubic structure, from equation (2) we can get the interaction energy density $\epsilon_{\rm I}$ as the function of the distance between two nearby H-clusters R,

$$\epsilon_{\rm I} = \frac{1}{2}n\left(A_1\frac{g_{\omega \rm H}^2}{4\pi}\frac{{\rm e}^{-m_\omega^* R}}{R} - A_2\frac{g_{\sigma \rm H}^2}{4\pi}\frac{{\rm e}^{-m_\sigma^* R}}{R}\right), \qquad (3)$$

where $A_1 = 6.2$ and $A_2 = 8.4$ are the coefficients from accounting all of the clusters' contributions (Huang & Han 1988). The number density of H-clusters n is $n = R^{-3}$, so $\epsilon_{\rm I}$ can be written as the function of n,

$$\epsilon_{\rm I} = \frac{1}{2}n^{4/3}\left(A_1\frac{g_{\omega \rm H}^2}{4\pi}{\rm e}^{-m_\omega^* n^{-1/3}} - A_2\frac{g_{\sigma \rm H}^2}{4\pi}{\rm e}^{-m_\sigma^* n^{-1/3}}\right). \qquad (4)$$

Considering the Brown–Rho scaling law for H-particles in equation (1), the rest-mass energy density also depends on the number density n of H-clusters, and the total energy density is then

$$\epsilon = \epsilon_{\rm I} + n m_{\rm H}^*, \qquad (5)$$

and the pressure is

$$P = n^2\frac{{\rm d}}{{\rm d}n}\left(\frac{\epsilon}{n}\right). \qquad (6)$$

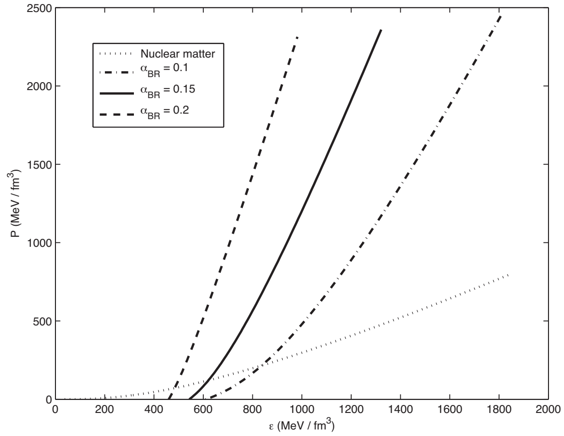

Figure 4. The equation of state for nuclear matter (Niu & Gao 2010) (dotted line), and H-cluster matter with $\alpha_{\rm BR} = 0.1$ (dash–dotted line), $\alpha_{\rm BR} = 0.15$ (solid line) and $\alpha_{\rm BR} = 0.2$ (dashed line). The density ranges are the same as in Fig. 2.

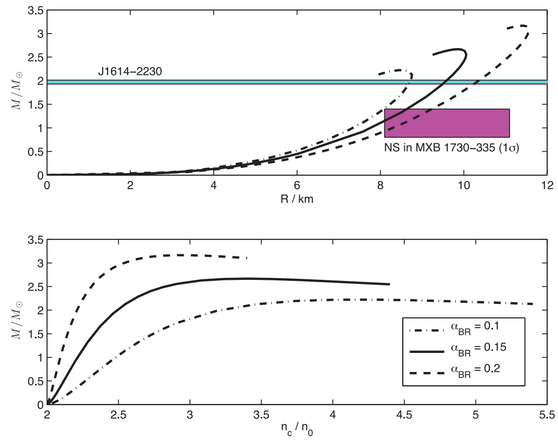

Figure 5. The mass–radius curves and mass–central density (rest-mass energy density) curves, in the case $n_{\rm s} = 2n_0$, including $\alpha_{\rm BR} = 0.1$ (dash–dotted line), $\alpha_{\rm BR} = 0.15$ (solid line) and $\alpha_{\rm BR} = 0.2$ (dashed line). The blue line (region) shows the mass of PSR J1614−2230 (Demorest et al. 2010), and the red rectangle shows the mass and radius (1σ) of the neutron star in MXB 1730−335 (Sala et al. 2012).

At the surface of a star the pressure should be vanishing, $P(n = n_{\rm s}) = 0$. Taking $g_{\omega{\rm H}}/g_{\omega{\rm N}} = 2$[3] (Faessler et al. 1997), $g_{\sigma{\rm H}}$ could be derived if we know the surface number density of H-clusters $n_{\rm s}$ (or the corresponding surface mass density $\rho_{\rm s}$).

The equations of state of H-cluster matter for different cases ($\alpha_{\rm BR} = 0.1, 0.15$ and 0.2) are shown in Fig. 4. To make comparison, we also show the equation of state of nuclear matter (Niu & Gao 2010). It is clear that the equation of state of H-cluster matter is much stiffer than that of nuclear matter. The stiffer equation of state leads to higher maximum mass, as will be shown in next subsection. The sound speed of H-cluster matter with such stiff equation of state will be discussed in Section 4.1.

Compact stars composed of pure H-clusters are electric neutral, but in reality there could be some flavour symmetry breaking that leads to the non-equality among *u*, *d* and *s*, usually with less *s* than *u* and *d*. The positively charged quark matter is necessary because it allows the existence of electrons that is crucial for us to understand the radiative properties of pulsars. The pressure of degenerate electrons is negligible compared to the pressure of H-clusters, so the contribution of electrons to the equation of state is negligible.

3.2 Mass–radius relation of H-cluster stars

In general relativity, the hydrostatic equilibrium condition in spherically symmetry is (Oppenheimer & Volkoff 1939)

$$\frac{1-2Gm(r)/c^2r}{P+\rho c^2}r^2\frac{{\rm d}P}{{\rm d}r}+\frac{Gm(r)}{c^2}+\frac{4\pi G}{c^4}r^3P=0, \quad (7)$$

where

$$m(r)=\int_0^r \rho 4\pi r'^2 {\rm d}r', \quad (8)$$

with $\rho = \epsilon_1/c^2 + m_{\rm H}^* n$. $m_{\rm H}^*$ is the in-medium value for the mass of H-clusters, which obeys the same scaling law as in equation (1). Inserting the equation of state $P(\rho)$ we can get the total mass M and radius R of an H-cluster star by numerical integration. Fig. 5 shows the mass–radius and mass–central density (rest-mass energy density) curves, in the case $n_{\rm s} = 2n_0$, including $\alpha_{\rm BR} = 0.1$, 0.15 and 0.2. At first, M grows larger as central density increases, and eventually M reaches the maximum value, after which the increase of central density leads to gravitational instability. In the figure, all the curves have maximum masses higher than $2\,{\rm M}_\odot$.

The observed masses of pulsars put constraints on the state of quark matter. Quark stars have been characterized by soft equations of state, because in conventional quark star models (e.g. MIT bag model) quarks are treated as relativistic and weakly interacting particles. Radio observations of a binary millisecond pulsar PSR J1614−2230 imply that the pulsar mass is $1.97 \pm 0.04\ {\rm M}_\odot$ (Demorest et al. 2010), shown in Fig. 5 as the blue region. Although we could still obtain high maximum masses under the MIT bag model by choosing suitable parameters (Zdunik et al. 2000), a more realistic equation of state in the density-dependent quark mass model (e.g. Dey et al. 1998) is very difficult to reach a high enough maximum stellar mass, which was considered as possible negative evidence for quark stars (Cottam, Paerels & Mendez 2002). Nevertheless, some other models of stars with quark matter could be consistent with the observation of the high-mass pulsar, such as colour-superconducting quark matter model (Ruster & Rischke 2004) and hybrid star models (Baldo et al. 2003; Alford 2008). Moreover, quark-cluster stars could also have maximum mass $M_{\rm max} > 2\,{\rm M}_\odot$ because of stiff equation of state (Lai & Xu 2009a,b, 2011). Recently, the mass and radius of the neutron star in the rapid burster MXB 1730−335 have been constrained to be $M = 1.1 \pm 0.3\,{\rm M}_\odot$ and $R = 9.6 \pm 1.5$ km (1σ) by the analysis of *Swift*/XRT time-resolved spectra of the burst (Sala et al. 2012), and this result is also shown in Fig. 5 as the red rectangle. Our results are consistent with both observations (at least in 2σ).

The real state of matter at densities of compact stars is essentially a non-perturbative QCD problem and thus difficult to solve. We make a phenomenological model that quarks could be grouped into quark clusters at this energy scale to propose the quark-cluster stars,

[3] The relation between $g_{\omega{\rm H}}$ and $g_{\omega{\rm N}}$ is in fact unknown, but the ratio of the two quantities $g_{\omega{\rm H}}/g_{\omega{\rm N}}$ seems crucial in our calculations. If the ratio is not large enough, e.g. if we choose the ratio to be 4/3 (as used in Glendenning & Schaffner-Bielich 1998), $g_{\sigma{\rm H}}$ would become negative in some cases (e.g. $\alpha_{\rm BR} = 0.15$ and $n_{\rm s} \leqslant 2.2\,n_0$). This is because the rest-mass energy density, also depending on the number density, leads to a corresponding negative pressure. How to choose the value of $g_{\omega{\rm H}}/g_{\omega{\rm N}}$ is still in controversy, and here we only choose one possible value to do our calculations.

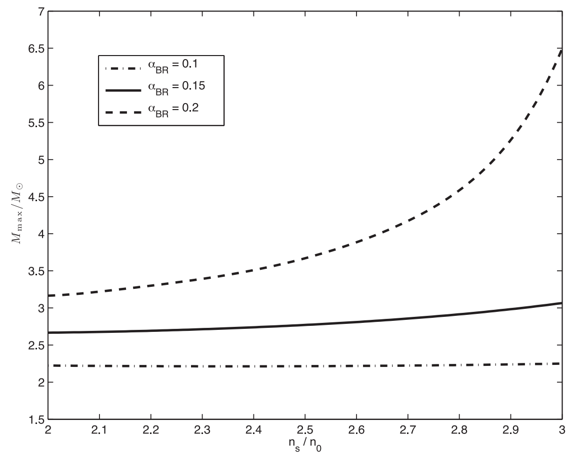

Figure 6. The dependence of $M_{\rm max}$ on surface density, in the cases $\alpha_{\rm BR} = 0.1$ (dash–dotted line), $\alpha_{\rm BR} = 0.15$ (solid line) and $\alpha_{\rm BR} = 0.2$ (dashed line).

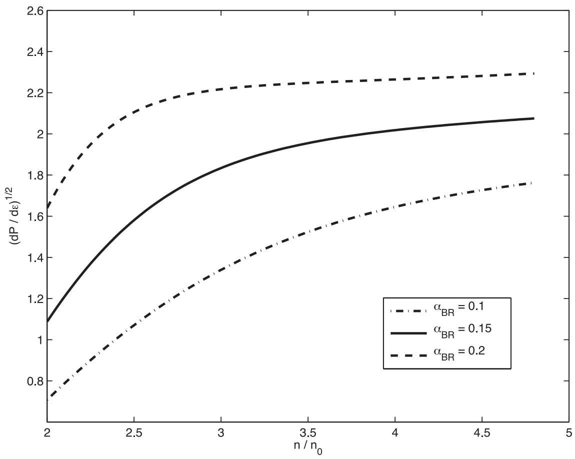

Figure 7. The values of $\sqrt{{\rm d}P/{\rm d}\epsilon}$ inside stars, in the cases $\alpha_{\rm BR} = 0.1$ (dash–dotted line), $\alpha_{\rm BR} = 0.15$ (solid line) and $\alpha_{\rm BR} = 0.2$ (dashed line).

and specify quark clusters in this paper to be H-clusters. The CSC model is the most often used one for modelling quark matter, but it is still uncertain that whether the interaction between quarks is weak enough to make the non-perturbative treatment to be reasonable. The point we consider in this paper is that, under the assumption of light flavour symmetry, H-clusters could be the possible kind of quark clusters, and as a specific quark-cluster stars, H-cluster stars could not be ruled out by the observed high-mass pulsars.

3.3 Maximum mass of H-cluster stars

We constrain $\alpha_{\rm BR}$ in the context of H-cluster stars by the maximum mass of pulsars $M_{\rm max}$, shown in Fig. 6. The interaction between H-dibaryons was studied previously and the related parameters were derived by fitting data in experiments of nucleon–nucleon interaction and hypernucleus events (e.g. see Sakai et al. 1997, and references therein); however, whether the two-particle interaction data are adequate in determining the properties of quark matter is uncertain. Our model for the H–H interaction could provide us another way to study the properties of H-clusters in quark matter, although giving wide ranges of parameters due to the uncertainty of $M_{\rm max}$. Fig. 6 shows the dependence of $M_{\rm max}$ on $\alpha_{\rm BR}$, in the cases $\alpha_{\rm BR} = 0.1$, 0.15 and 0.2. When $\alpha_{\rm BR} \leq 0.15$, the discrepancy between different values of surface densities is not significant, and under the range of parameter space we choose here $M_{\rm max}$ can be well above $2\,{\rm M}_\odot$.

We derive the maximum mass of H-cluster stars to show that they could have safe maximum mass high enough to accord with the observations, although there are certainly some other kind of quark star models which provide possible ways to explain the observed high mass of the newly discovered pulsar PSR J1614−2230. However, it should be noted that the highest mass of pulsars that we find is surely different from the real (theoretical) maximum mass that a stable pulsar is able to have against gravity. But how to infer the real maximum mass from the observed masses? The relation between the two in the case of pulsars could be compared to the case of white dwarfs, if we assume that the observed mass of pulsars and white dwarfs (or in fact any other objects) could have the same bias towards their real maximum mass due to observational effects. Here we choose white dwarfs to make the comparison since the maximum mass of white dwarfs is well established to be about $1.4\,{\rm M}_\odot$.

The statistical study of nearby white dwarfs lying within 20 pc of the Sun shows that the distribution of measured masses of such a sample of white dwarfs has a peak at around $0.6\,{\rm M}_\odot$ and the most massive one is about $1.3\,{\rm M}_\odot$ (Kepler et al. 2007; Giammichlel, Bergeron & Dufour 2012). Assuming the same scaling of measured masses and the real maximum mass for the case of pulsars, whose distribution of measured masses shows a peak at around $1.4\,{\rm M}_\odot$ (Valentim, Rangel & Horvath 2011; Zhang et al. 2011), we could infer that the maximum mass of pulsars can be estimated to be $\sim 3.3\,{\rm M}_\odot$ (using the peak value) or $\sim 2.2\,{\rm M}_\odot$ (using $2\,{\rm M}_\odot$ as the maximum value). The estimated maximum mass for pulsar-like stars would be $\sim 3\,{\rm M}_\odot$, which is still much lower than the detected minimal mass ($\sim 5\,{\rm M}_\odot$) of stellar black holes (Bailyn et al. 1998), if the mass ($2.74\,{\rm M}_\odot$) of a pulsar (J1748−2021B) in a globular cluster is confirmed in the future. As shown by our results, H-cluster stars are consistent with the above estimation because their maximum mass could be $\sim 3\,{\rm M}_\odot$ or even higher (e.g. $\alpha_{\rm BR} \geq 0.15$). Therefore, discovering more massive pulsars in the future will certainly be helpful for us to get closer to the maximum mass and distinguish different models.

4 DISCUSSIONS

4.1 About the stiff equation of state

Composed of non-relativistic H-clusters with an interaction in the form of equation (2), quark-cluster stars could have a stiff equation of state and a high maximum mass. Under a wide range of densities, showing in Fig. 7, we have $\sqrt{{\rm d}P/{\rm d}\epsilon} > 1$. If we treat the sound speed (i.e. the signal propagation speed) as $c_{\rm s} = \sqrt{{\rm d}P/{\rm d}\epsilon}$, then we will have the 'superluminal' sound speed. However, whether the real sound speed can always been calculated as $c_{\rm s}$ may depend on the structure of matter and the way we introduce the interparticle interaction. The probability that $c_{\rm s}$ exceeds the speed of light in ultradense matter was studied previously by Bludman & Ruderman (1968). Caporaso & Brecher (1979) then proposed explicitly that, in a lattice with $P(\epsilon)$ relation arising from a static calculation, $c_{\rm s}$ is not a dynamically meaningful speed, i.e. $c_{\rm s}$ is not a signal propagation speed.

Causality is a very basic property of dense matter theory, which is widely accepted. In the case of fluid, the real sound speed is calculated as $c_{\rm s}$, so causality is violated when ${\rm d}P/{\rm d}\epsilon > 1$. However, the situation is much more complicated when translational symmetry breaks for solid quark-cluster matter where quarks are clustered in lattice. Moreover, we use a classical potential model, i.e. a kind of action at a distance, to derive the total energy density and thus the equation of state. In this non-relativistic model, $P(\epsilon)$ may only measure the stiffness of equation of state, and the real sound speed is too difficult to calculate. From the picture of the mechanism for interaction in our model, the interaction is mediated by mesons, so the real speed of interaction is obviously smaller than the speed of light, i.e. the signal propagation speed remains subluminal.

Our equation of state is very stiff because of two main reasons. First, quark clusters are non-relativistic particles. For the system composed of non-relativistic particles, ignoring the interaction, the momentum of each particle could be approximated as $p \propto n^{1/3}$ (from Heisenberg's relation) and the total energy density as $\epsilon \propto n$. The kinetic energy per particle $E_{\rm k} \propto p^2$, and then the kinetic energy density $\epsilon_{\rm k} \propto np^2 \propto n^{5/3}$, so pressure $P = n^2\partial(\epsilon_{\rm k}/n)/\partial n \propto n^{5/3} \propto \epsilon^{5/3}$. On the other hand, for extremely relativistic particles, from the same estimation, $P \propto \epsilon$. Therefore, the equation of state of the non-relativistic system is stiffer than that of relativistic one. Secondly, each quark cluster is trapped inside the potential well formed by the neighbouring quark clusters, and the in-medium stiffening effect makes the shape of potential well to be stiffer. From the above arguments, our model is different from the mainstream that quark stars, composed of relativistic and weakly interacting quarks, are characterized by a soft equation of state. Phenomenologically, a corresponding-state approach to quark-cluster matter (Guo, Lai & Xu 2012) also results in a very stiff equation of state.

4.2 The binding of H-clusters inside stars

At the highest density of the stars, with $n_{\rm B} \simeq 8.3n_0$, the distance between two nearby H-clusters is about 1.1 fm. The size of H-dibaryons could be a little larger than that of nucleons, and then at the centre of the star they could be so crowded that they touch the nearby ones, but they should be safe against being crushed. We assume that the touch of nearby H-clusters does not influence our overall picture, since it only happens at the very centre of the star and the degree of touch is not high to cause dissociation.

In fact, the dependence of binding energy of quark clusters on density is still unknown. However, if the mass of H-dibaryons decreases with increasing densities like baryons and mesons, this could be equivalent to the increase of binding energy of H-dibaryons. At densities beyond ρ_0, the degrees of freedom become complex due to the non-perturbative nature of QCD, which could be responsible for the different binding behaviour to the case at densities below ρ_0.

Conventional strange quark matter (without quark clustering) is thought to be stable in bulk but unstable in the case of light strangelets when the baryon number A is as small as 6 (see a review in Madsen 1999). Although quark clusters are similar to light strangelets, they could still be stable as they are in a medium but not in vacuum. The highly dense environment makes the energy per baryon of H-clusters to be lower than that of neutron and nucleon matter, as has been shown above. As a kind of light strangelets, H-dibaryons are unstable in vacuum, which make it difficult to study them experimentally; however, inside compact stars, the in-medium effect could stabilize them.

4.3 H-cluster stars are self-bound

It is worth noting that, although composed of H-clusters, H-cluster stars are self-bound. They are bound by the interaction between quark clusters (the H-clusters here). This is different from but similar to the traditional MIT bag scenario. The interaction between H-clusters could be strong enough to bind the star, and on the surface, the quark clusters are just in the potential well of the interaction, leading to non-vanishing density but vanishing pressure.

It is surely possible that there could be normal matter surrounding a self-bound H-cluster star, but initially the surroundings would not remain because of energetic exploding (Ouyed, Rapp & Vogt 2005; Paczynski & Haensel 2005; Chen, Yu & Xu 2007).

4.4 Clustering quark matter

Quark clusters could emerge in cold dense matter because of the strong coupling between quarks. The quark-clustering phase has high density and the strong interaction is still dominant, so it is different from the usual hadron phase. On the other hand, the quark-clustering phase is also different from the conventional quark matter phase which is composed of relativistic and weakly interacting quarks. The quark-clustering phase could be considered as an intermediate state between the hadron phase and the free-quark phase, with deconfined quarks grouped into quark clusters, and this is another reason why we take quark-cluster stars as a special kind of quark stars. H-cluster stars are self-bound due to the interaction between clusters, with non-vanishing surface density but vanishing surface pressure.

Whether the chiral symmetry broken and confinement phase transition happen simultaneously inside compact stars is a matter of debate (see Andronic et al. 2010, and references therein), but here we assume that the chiral symmetry is broken in the quark-clustering phase.

4.5 From the asymmetry term to a flavour symmetry

It is well known that there is an asymmetry term to account for the observed tendency to have equal numbers of protons (Z) and neutrons (N) in the liquid drop model of the nucleus. This nuclear symmetry energy (or the isospin one) represents a symmetry between proton and neutron in the nucleon degree of freedom, and is actually that of up and down quarks in the quark degree (Li, Chen & Ko 2008). The possibility of electrons inside a nucleus is negligible because its radius is much smaller than the Compton wavelength $\lambda_{\rm c} = h/m_{\rm e}c = 0.24$ Å. The lepton degree of freedom would then be not significant for nucleus, but what if the nuclear radius becomes larger and larger (even $\gg \lambda_{\rm c}$)?

Electrons are inside a large or gigantic nucleus, which is the case of compact stars. Now there is a competition: isospin symmetry favours $Z = N$, while lepton chemical equilibrium tends to have $Z \ll N$. The nuclear symmetry energy $\sim 30(Z-N)^2/A$ MeV (at saturated nuclear matter density ρ_0), where $A = Z + N$, could be around 30 MeV per baryon if $N \gg Z$. Interestingly, the kinematic energy of an electron is $\sim$100 MeV if the isospin symmetry holds in nuclear matter. However, the situation becomes different if strangeness is included: no electrons exist if the matter is composed by equal numbers of light quarks of u, d and s with chemical equilibrium. In this case, the three-flavour symmetry, an analogy of the symmetry of u and d in nucleus, may result in a ground state of matter for gigantic nuclei. Certainly, the mass difference between u, d and s quarks would also break the symmetry, but the interaction between quarks could lower the effect of mass differences and try to restore the symmetry. Although it is difficult for us to calculate how strong the interaction between quarks is, the non-perturbative nature and the energy scale of the system make it reasonable to assume that the degree of the light flavour symmetry breaking is small, and there is a few electrons (with energy ∼10 MeV). Heavy flavours of quarks (*c*, *t* and *b*) could not be existed if cold matter is at only a few nuclear densities.

The above argument could be considered as an extension of the Bodmer–Witten conjecture. Possibly it does not matter whether three flavours of quarks are free or bound. We may thus re-define *strange matter* as cold dense matter with light flavour symmetry of three flavours of *u*, *d* and *s* quarks.

5 CONCLUSIONS

We propose in this paper that the strong interaction between quarks inside compact stars renders quarks grouped into a special kind of quark clusters, H-clusters, leading to the formation of H-cluster stars. Although there are many uncertainties about the stability of H-cluster matter, it could be possible that at high densities H-cluster matter is stable against transforming to nucleon matter.

The equation of state of H-cluster stars is derived by assuming the Yukawa form of H–H interaction under meson exchanges, and the in-medium effect from the Brown–Rho scaling law of meson masses is also taken into account. H-cluster stars could have a stiff equation of state, and under a wide range of parameter space, the maximum mass of H-cluster stars can be well above 2 $M_\odot$. Furthermore, if we know about the properties of pulsars from observations, we can get information on the H–H interaction; for example, if a pulsar with mass larger than 3 $M_\odot$ is discovered, then we can constrain the coefficient of Brown–Rho scaling $\alpha_{\rm BR} \geq 0.15$.

Although the state of cold quark matter at a few nuclear densities is still an unsolved problem in low-energy QCD, various pulsar phenomena would give us some hints about the properties of an elemental strong interaction (Xu 2010), complementary to the terrestrial experiments. Pulsar-like compact stars provide high-density and relatively low temperature conditions where quark matter with H-clusters could exist. H-cluster has been the subject of many theoretical and experimental studies. It is in controversy that whether H-cluster is a bound state, which depends on the quark masses (Shanahan, Thomas & Young 2011), and the binding behaviour at high densities is still unknown. Whether quark matter composed of H-clusters could achieve supranuclear density is still uncertain, and on the other hand, the nature of pulsar-like stars also depends on the physics of dense matter. These problems are essentially related to the non-perturbative QCD, and we hope that future astrophysical observations would test the existence of H-cluster stars.

Finally, we would clarify two questions and answers, which should be beneficial to make sense about the conclusions presented in this paper. (1) *Why does not an H-particle on the surface decay into nucleons?* The reason could be similar to that why a neutron does not decay into proton in a stable nucleus. Landau (1938) demonstrated that significant gravitational energy would be released if neutrons are concentrated in the core of a star; it is now recognized, however, that the fundamental colour interaction is more effective and stronger than gravity to confine nucleons. The equality condition of chemical potentials at the boundary between two phases applies to gravity-confined stars (Landau 1938), but may not to self-bound objects by a strong interaction. (2) *Why can hardly normal matter be converted into more stable H-cluster matter in reality?* We know that ^{56}Fe is most stable nucleus, but it needs substantial thermal kinematic energy to make nuclear fusion of light nuclei in order to penetrate the Coulomb barrier. Strong gravity of an evolved massive star dominates the electromagnetic force, compressing baryonic matter into quark-cluster matter in astrophysics. This is expensive and rare.

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