

# Monopole-charged Pulsars and Relevant Issues

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**Abstract** An *aligned* pulsar whose rotation axis and magnetic dipole axis are parallel should be positively charged. We consider the electromagnetic field both inside and outside the star under a specific condition and derive the total charge of a pulsar. The statistical relation between the pulsar's rotation energy loss rate (or the period derivative) and the period may imply that the millisecond radio pulsars with small periods could be low-mass bare strange stars.

**Key words:** pulsars: general — stars: neutron — dense matter

## 1 INTRODUCTION

Study of the pulsar magnetosphere is essential for us to understand various radiative processes, and thus observations in different bands. The charge-separated plasma (Goldreich and Julian 1969) in the pulsar's magnetosphere of various gap models (Ruderman and Sutherland 1975; Arons and Scharlemenn 1979; Cheng, Ho and Ruderman 1986; Qiao et al. 2004) has been investigated.

In the recent study of the electromagnetic field of an *aligned* pulsar whose rotation axis is parallel to the magnetic axis, Xu et al. (2006) suggested that these pulsars should be positively monopole-charged and that the millisecond radio pulsars with small periods could be low-mass bare strange stars.

## 2 PULSAR'S NET CHARGE

If neutron star is assumed to be magnetized homogenously, its magnetic field inside and outside the star can be described as

$$\mathbf{B}_{\text{in}} = \frac{8\pi}{3}M = \frac{2\mathbf{m}}{r_0^3} \quad (r < r_0), \quad (1)$$

and

$$\mathbf{B}_{\text{out}} = \frac{3\hat{r}(\hat{r} \cdot \mathbf{m}) - \mathbf{m}}{r^3} \quad (r > r_0), \quad (2)$$

where  $M$  is the magnetized intensity,  $\mathbf{m}$  is the magnetic dipole moment,  $r_0$  is star's radius,  $r$  is the radial distance to the star's centre, and  $\hat{r}$  is the radial direction. Here we define  $|\mathbf{B}_{\text{in}}| \equiv B_0$ .

If all the particles both inside and outside the star corotate with respect to the rotation axis which is parallel to the magnetic axis, the electric field,  $\mathbf{E}$ , satisfies

$$\mathbf{E} + \frac{\boldsymbol{\Omega} \times \mathbf{r}}{c} \times \mathbf{B} = 0, \quad (3)$$

where we ignore the magnetic field induced by the corotating particles,  $\boldsymbol{\Omega} = 2\pi/P$  is the pulsar's angular velocity and  $P$  is the pulsar period. If  $\hat{k}$  is the unit vector of angular velocity,  $\boldsymbol{\Omega} = \Omega\hat{k}$ . And for *aligned* pulsars, the direction of the magnetic dipole moment is the same as that of the angular velocity, i.e.,  $\mathbf{m} = m\hat{k}$ . We estimate the charge under this specific condition.

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From Equations (1) to (3), the electric fields in and out a pulsar are, respectively,

$$\mathbf{E}_{\text{in}} = \frac{\Omega r B_0}{c} \hat{k} \times (\hat{k} \times \hat{r}) \quad (r < r_0), \quad (4)$$

$$\mathbf{E}_{\text{out}} = \frac{\Omega r_0^3 B_0}{2cr^2} [3(\hat{k} \cdot \hat{r})\hat{r} - \hat{k}] \times (\hat{k} \times \hat{r}) \quad (r > r_0). \quad (5)$$

The corresponding charges then could be

$$Q_{\text{in}} = \rho_{\text{in}} \frac{4\pi r_0^3}{3} = \frac{\nabla \cdot \mathbf{E}_{\text{in}}}{4\pi} \frac{4\pi r_0^3}{3} = -\frac{2r_0^3 \Omega B_0}{3c}, \quad (6)$$

and  $Q_{\text{out}} = \int \rho_{\text{out}} \cdot dV = 0$ , where  $\rho_{\text{in}}$  and  $\rho_{\text{out}}$  are the charge densities in and out the star. From Equations (4) and (5), the charge intensity and the charges on the stellar surface are

$$\sigma_s = \frac{1}{4\pi} (\mathbf{E}_{\text{out}} - \mathbf{E}_{\text{in}})_{r=r_0} \cdot \hat{r} = \frac{3\Omega r_0 B_0 \sin^2 \theta}{8\pi c}, \quad (7)$$

$$Q_s = \int_{r=r_0} \sigma_s \cdot ds = \frac{r_0^3 \Omega B_0}{c}, \quad (8)$$

where  $\theta$  is the polar angle between  $\hat{r}$  and  $\hat{k}$ . From the above equations, we can obtain the total charges

$$Q_{\text{total}} = Q_{\text{in}} + Q_{\text{out}} + Q_s = \frac{2\pi r_0^3 B_0}{3Pc} \simeq 2.3 \times 10^{10} \frac{r_6^3 B_{12}}{P} \quad (\text{coulombs}), \quad (9)$$

where  $r_6 = r_0/(10^6 \text{ cm})$ ,  $B_{12} = B_0/(10^{12} \text{ G})$ . Thus the aligned pulsars should be positively charged.

It is evident that this total charges in Equation (9) is different from that obtained in Equation (17) of Xu et al. (2006,  $Q \sim 10^{-3} r_6^3 B_{12}/P^2$ ). This difference could result in a shift of the boundary (i.e., the field lines being at the same electric potential as those in the interstellar medium) which separating regions I and II in figure 1 of Xu et al. (2006): the boundary should be closer (i.e., above the critical field lines) to the magnetic axis if the real charge is greater than  $Q$ . Since the global solution to pulsar magnetosphere is not available, it is impractical to give an accurate estimate of the pulsar's net charge.

### 3 EVIDENCE FOR LOW-MASS MILLISECOND PULSARS

There are 1394 radio pulsars<sup>1</sup> with known  $P$  and  $\dot{P}$  (thus  $\dot{E}$ ) simultaneously. The numbers of millisecond and normal radio pulsars are 87 and 1307 if one assumes the dividing line is  $B_c = 6.4 \times 10^{19} \sqrt{P\dot{P}}$  G =  $5 \times 10^{10}$  G as shown in left upper panel of Figure 1. Here we only consider 87 millisecond pulsars ( $B < B_c$ ).

It is known that the numerical value of rotation energy loss rate

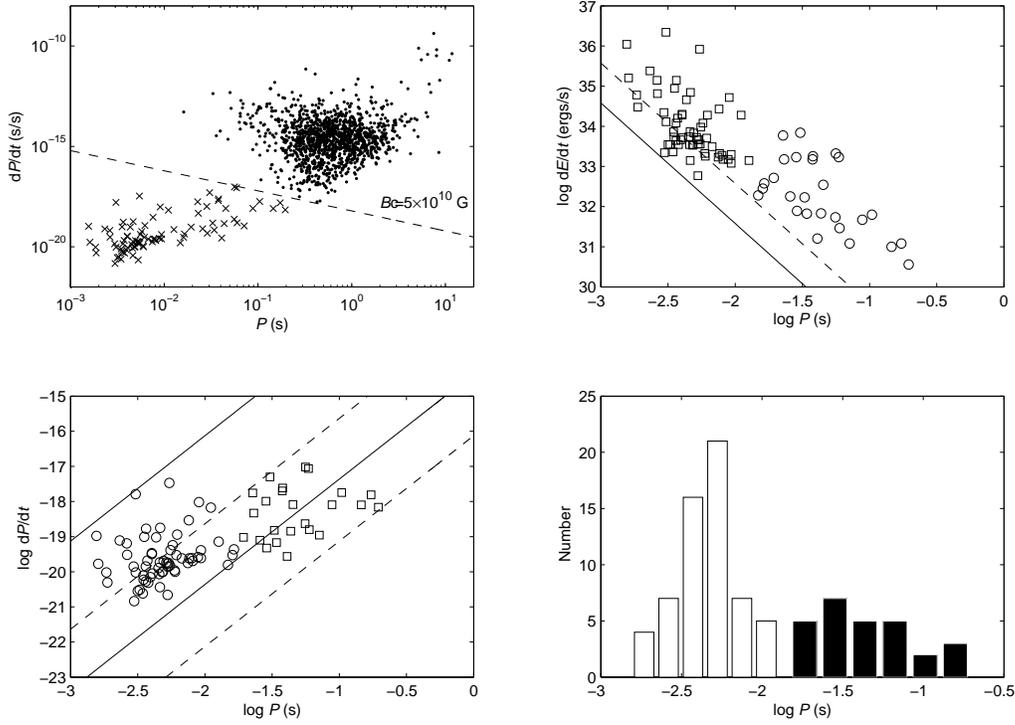
$$\dot{E} = -I\Omega\dot{\Omega} \simeq 3.1 \times 10^{25} r_6^2 \dot{P}_{-20} P^{-3}, \quad (10)$$

where rotational inertia of star  $I \simeq 2Mr_6^2/5$  (the star is assumed to be a homogeneous rigid sphere),  $\dot{P}_{-20} = \dot{P}/10^{-20}$ . Note that the star's radius is considered as a variable. A potential drop across the open field line region for a rotating magnetic pulsar could be simply

$$\Phi = 6.6 \times 10^{12} B_{12} r_6^3 P^{-2} \quad \text{V}. \quad (11)$$

To infer the star's radius and to show possible evidence for low-mass millisecond radio pulsars in the bare strange star model by observational data, we firstly investigate the relations of  $\dot{E} - P$  and  $\dot{P} - P$  as shown in left lower panel and right upper panel of Figure 1. Secondly, applying the K-means method, we divide them into two groups: 60 pulsars in *Group I* and 27 pulsars in *Group II*; 63 in *Group I'* and 24 in *Group II'*. Thirdly, from  $B = 6.4 \times 10^{19} \sqrt{P\dot{P}}$  G and assigning  $\dot{P}_{-20} = 1.0 \text{ s/s}$ ,  $\Phi = 1.0 \times 10^{12} \text{ V}$  in Equations (10) and (11) for 87 millisecond pulsars, we give the lower limit to radius for these two groups  $r_{6,I} \approx 0.35$  and  $r_{6,II} \approx 1.1$  and the limit ranges of radii  $0.065 \leq r_{6,I'} \leq 0.35$  and  $0.17 \leq r_{6,II'} \leq 0.65$ . Finally, from the analysis above and according to Equation (10) in Xu (2005), we can obtain the radius and mass ratios of *Group I* to *Group II* from the relation of  $P - \dot{E}$  and those of *Group I'* to *Group II'* from the relation of  $P - \dot{P}$  as shown in Table 1.

<sup>1</sup> <http://www.atnf.csiro.au/research/pulsar/psrcat/>



**Fig. 1** Left upper panel: Period derivatives  $\dot{P}$  as a function of period  $P$  for 1349 radio pulsars. The dots represent the normal radio pulsars with  $B > B_c$  and the crosses are the millisecond radio pulsars with  $B < B_c$ , where the dividing line is the magnetic field  $B_c = 5 \times 10^{10}$  G. Left lower panel: The period derivative  $\dot{P}$  as a function of period  $P$  for millisecond radio pulsars. Circles (*Group I*) and squares (*Group II*) are two groups divided by K-means methods. The regions between two solid lines (for *Group I*) and dash lines (*Group II*) represent limit ranges of radii for these two groups, respectively. Right upper panel: Spin down energy loss rate  $\dot{E}$  as a function of period  $P$  for millisecond radio pulsars. The empty circles (*Group I*) and squares (*Group II*) are two groups divided by K-means methods. The solid and dash lines represent the minimums of radius for these two groups. Right lower panel: Two-peak period  $P$  distribution of 87 millisecond radio pulsars.

**Table 1** The radius and mass ratios of *Group I* to *Group II* from the relation of  $P - \dot{E}$  and those of *Group I*' to *Group II*' from the relation of  $P - \dot{P}$ .

Relation	Radius Ratio	Mass Ratio
$P - \dot{E}$	0.32	0.03
$P - \dot{P}$	0.01	0.13

## 4 CONCLUSIONS

The total charge of a uniformly magnetized aligned pulsar with the Goldreich-Julian density outside the star is calculated. The boundary which separates the positive and negative flows in a magnetosphere is not necessarily those critical lines if the net charge is not that as assumed by Xu et al. (2006). The statistical relation between the pulsar's  $\dot{E}$  (or the period derivative  $\dot{P}$ ) and the period  $P$  may indicate that the millisecond radio pulsars with small periods are low-mass bare strange stars.

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