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Pulsar slow glitches in a solid quark star model

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ABSTRACT

A series of five unusual slow glitches of the radio pulsar B1822–09 (PSR J1825–0935) was observed between 1995 and 2005. This is a phenomenon that is understood in a solid quark star model, and reasonable parameters for slow glitches are given in this paper. We propose that, because of increasing shear stress as the pulsar spins down, a slow glitch may occur, beginning with the collapse of a superficial layer of the quark star. This layer of material turns to viscous fluid at first, the viscosity of which helps to deplete the energy released from both the accumulated elastic energy and the gravitation potential. There is then a slow glitch. Numerical calculations show that the slow glitches that have been observed could be reproduced if the effective coefficient of viscosity is $\sim 10^2$ cm^2 s^{-1} and the initial velocity of the superficial layer is of the order of 10^{-10} cm s^{-1} in the coordinate rotating frame of the star.

Key words: dense matter – stars: neutron – pulsars: general – pulsars: individual: PSR B1822–09.

1 INTRODUCTION

It is a pity that, even 40 yr after the discovery of pulsars, we are not yet able to determine confidently what state of matter really exists in pulsar-like stars, because of the difficulty of calculation using the first principles of the elementary strong interaction. From the time of Landau, nuclear matter (related to neutron stars) has been considered one of the possibilities, while quark matter (related to quark stars) is an alternative, because of the asymptotic freedom of the strong interaction between quarks (for example, see the following reviews Madsen 1999; Glendenning 2000; Lattimer & Prakash 2004; Weber 2005). It is necessary to focus on astrophysical observations in order to solve this important and fundamental question.

Actually, pulsars are ideal astro-laboratories to study the physics of cold matter at supranuclear density. Based on a Planck-like spectrum without atomic features and the precession properties of pulsar-like stars, it has been conjectured that quark matter could be in a solid state (Xu 2003). In addition, other features naturally explained within this model could possibly include subpulse drifting in radio emission (Xu, Qiao & Zhang 1999), glitches (Zhou et al. 2004), strong magnetic fields, birth after a successful core-collapse supernova, and detection of small bolometric radii (for a short review, see, for example, Xu 2006). Moreover, the observational features of anomalous X-ray pulsars/soft gamma-ray repeaters may also reflect the nature of solid quark stars (Horvath 2005; Owen 2005; Xu, Tao & Yang 2006).

The slow glitches recently observed in PSR B1822–09 (Shabanova 1998; Zou et al. 2004; Shabanova 2005, 2007; Shabanova & Urama 2005) could be a new way to study cold solid quark matter. The pulsar has a period of 0.769 s and a relatively young age of 230 kyr. What is interesting about this pulsar are the changes in its period, known as slow glitches. Unlike the typical glitch phenomenon of other pulsars, this pulsar shows glitches that have a rather slow increase in spin frequency, with a time-scale of 200–300 d (while normal glitches experience this process much more swiftly, perhaps less than a spin period). Moreover, this pulsar does not experience any relaxation in the days following a slow glitch.

A series of five slow glitches are shown in Fig. 1. Fig. 1(a) shows the frequency derivative $\dot{\nu}$. The peaks of $\dot{\nu}$ are enveloped in a parabolic curve, which may indicate that all the slow glitches could be the components of one process. This process could be triggered by the small glitch ($\Delta\nu/\nu \sim 8 \times 10^{-10}$) that occurred in 1994 September. Both Figs. 1(b) and (c) show the frequency residual $\Delta\nu$, but relative to fits to data for different intervals (1991–1994 and 1995–2004, respectively). As a result of these observations, it is evident that the typical characteristic of a slow glitch is shown in Fig. 1(b); the spin frequency experiences a process that increases gradually but never decreases (i.e. there is no post-glitch relaxation).

We are trying to explain this phenomenon using the study of solid quark stars. Because the pulsar is supposed to be mostly built up of solid quark matter and the typical density there is of the order of 10^{14} g cm^{-3}, soon after a quake, the motion of only a thin layer of quark matter may cause a significant change in the moment of inertia (and thus the spin frequency) of the star, owing to its extremely high density. A numerical calculation is carried out to simulate the observational results of both the slow increase in the rotating frequency and the lack of post-glitch relaxation.

2 THE MODEL

We suggest that the star is a solid quark star with a typical density of 10^{14} g cm^{-3}. Because of magnetodipole radiation and the particle

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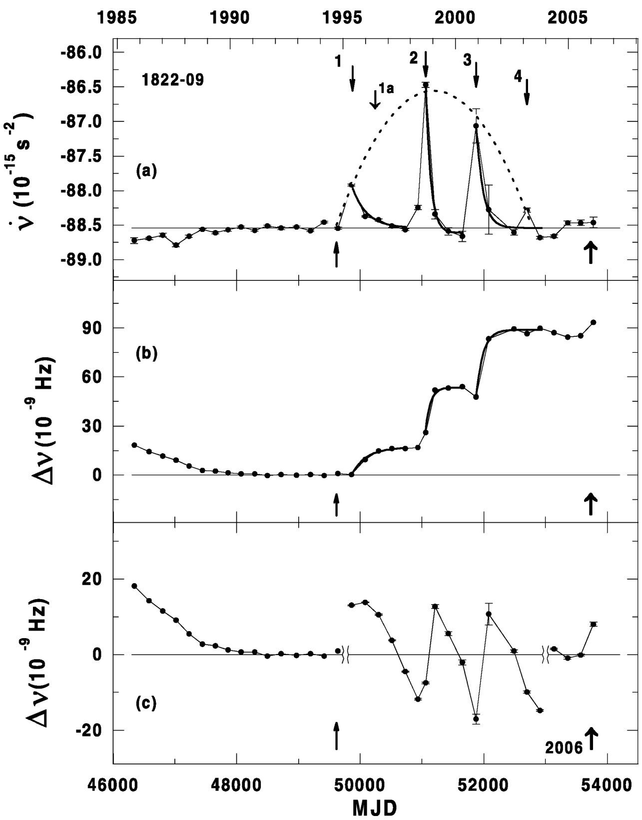

Figure 1. Observational data of the frequency residual $\Delta\nu$ and the frequency derivative $\dot{\nu}$ for the pulsar B1822–09. Arrows pointing downwards indicate the epochs at which five slow glitches occurred, while arrows pointing upwards indicate two normal glitches. The figure has been created by Shabanova (2007).

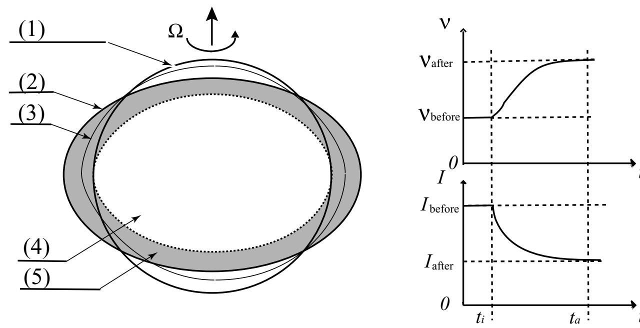

Figure 2. Schematic diagram of the model for slow glitches. The left side of this figure shows the change of the eccentricity of the star during a slow glitch, and the layer activity during this process. The right side of this figure shows the change of ν and I versus time. (Note that the usual change of ν and I as a result of the energy lost by magnetodipole radiation and particle emission is neglected here.) A slow glitch is supposed to occur at time $t_{\rm i}$. Left: (1) an imagined ellipsoidal figure (determined by equation 1) at $t_{\rm i}$ if the star is in a fluid state, with spin frequency $\nu = \nu_{\rm before}$; (2) the real figure of a solid star at $t_{\rm i}$ ($\nu = \nu_{\rm before}$), with stress energy high enough to quake; (3) the figure after a slow glitch at time $t_{\rm a}$, with $\nu = \nu_{\rm after}$, which might be close to the imagined shape (1); (4) the inner solid part which remains almost the same during a slow glitch; (5) the superficial crust layer, which is transmuted during the slow glitch.

ejection of the star, its rotation energy is lost and the frequency of the rotation declines. However, provided that the pulsar cools down from a hotter fluid ellipse, the equilibrium of the pulsar, with spin frequency ν, results in an eccentricity e:

$$\nu^2 = 8\pi^3 G\rho \left[\frac{\sqrt{1-e^2}}{e^3}(3-2e^2)\arcsin e - \frac{3(1-e^2)}{e^2} \right]. \qquad (1)$$

Here, G is the gravitational constant and ρ is the average density of the star (e.g. Shapiro & Teukolsky 1983). The configuration of equation (1) is for spinning stars made up of homogeneous fluid matter. Quark stars can be well approximated by uniform density if their masses are no greater than $\sim$1.5 $M_\odot$ (Alcock, Farhi & Olinto 1986). Equation (1) can be simplified for stars with small eccentricities as long as they do not spin very fast:

$$\nu = 4\pi e \sqrt{\frac{2\pi\rho G}{15}}. \qquad (2)$$

The relation of equation (2) implies that when the frequency of a star decreases, the eccentricity of the star should decrease accordingly in order to maintain the equilibrium shape if the star is in a fluid state. However, as the pulsar cools down and remains a solid star, elastic stress might occur and accumulate to resist the change in star shape. When this stress develops to a critical value that the material of the star cannot withstand, part of the star breaks (or collapses) and the elastic energy is then released. This is known as the quake of a solid quark star (Zhou et al. 2004; Xu 2006), during which both elastic and gravitational energies are released. In the model of Zhou et al. (2004), the moment of inertia, I, decreases suddenly (which results in a sharp increase $\Delta\nu$), and then increases gradually (to result in a following post-glitch relaxation). However, as the elastic force develops to a critical value, a solid rotator with a smaller shear modulus might not react so violently (i.e. I does not decrease suddenly), and I might decrease in a more gentle way, thus reproducing the feature of slow glitches that has been observed. No increase of I occurs if no significant elastic energy is converted to kinematic energy (and thus there is no post-glitch relaxation). The small glitch of PSR B1822–09 observed in 1994 may have led to a small effective shear modulus (e.g. a significant part of the quark matter may be shiver-like), and thus triggered the slow glitches. A pre-glitch could then be expected for slow glitches in the star-quake model of quark stars, in order to have an effective small shear modulus.

Soon after a quake, we assume that a superficial layer moves in order to set a new equilibrium (see Fig. 2), while most of the inner part of star remains almost the same. Some of the energy released turns into heat and melts the debris, while the rest turns to kinetic energy of the fluid. It is suggested in this model that the broken material will turn into viscous fluid, which is therefore able to flow slowly and change the shape of the star towards an equilibrium shape. This process would surely cause a slow decrease of the moment of inertia of the star. Because this process occurs over a short time compared to the typical age of a pulsar, it is reasonable to suppose that the angular momentum of the star remains invariant during the whole progress of the glitch. As a result, the frequency of the star might increase slowly at the same time. When the fluid is flowing, the viscous interaction among the elements of the fluid may exhaust the kinetic energy of the fluid. The shape of the star finally stops changing, and the material then cools down and is solidified. This is why a slow glitch appears to have no relaxation feature in the post-glitch period. Results from calculations (see Section 3) explain why this pulsar experiences a serious of five slow glitches rather than a single glitch.

To validate this model, we perform numerical calculations to simulate this process. First, the Navier–Stokes equation is known as the basic equation to describe the motion of viscous fluid. In the frame that is fixed on the inner solid part of the star, the general form of the equation is

$$\frac{\partial}{\partial t}\boldsymbol{v} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -\frac{1}{\rho}\nabla p + \mu\Delta\boldsymbol{v} + \boldsymbol{g} - \left[\frac{\mathrm{d}\omega}{\mathrm{d}t}\times\boldsymbol{r} + \omega\times(\omega\times\boldsymbol{r}) - 2\boldsymbol{v}\times\omega\right]. \quad (3)$$

Here, $\boldsymbol{v}$ is the velocity of superficial matter in the rotating frame, p is the pressure in the star, $\boldsymbol{g}$ is the acceleration of gravity on the surface of the star, μ denotes the viscosity of the fluid after the phase transformation (from solid to fluid-like matter), and the star is assumed to rotate constantly. The three terms in the brackets on the right-hand side arise from the inertial acceleration as a result of the non-inertial frame we have chosen.

Let us consider briefly equation (3). For a zero-order approximation, a solid star could be modelled by a rigidity body, with velocity $\boldsymbol{v} = 0$, that is,

$$0 = -\frac{1}{\rho}\nabla p + \boldsymbol{g} - \left[\frac{\mathrm{d}\omega}{\mathrm{d}t}\times\boldsymbol{r} + \omega\times(\omega\times\boldsymbol{r})\right]. \quad (4)$$

The above equation could at least be adaptable for the case of $v \ll r\omega$. As noted through observations, the time-scale of slow glitches is much longer than the spin period. Thus, we think that equation (4) can be applied in the following simulation. When a glitch occurs, the elastic energy is released suddenly, and the solid superficial layer turns to a fluid-like state with strong viscosity, but the pressure gradient ∇p may remain almost invariant. It is a key point that ∇p remains approximately the same in spite of the stress release in the model. This means that this is the case in which most of the elastic energy released is changed into heat, rather than into kinetic energy. The heating may melt down solid quark matter, which then becomes a viscous fluid. Soon after the starquake, the matter could resolidify finally as a result of cooling. In contrast, when the star breaks globally, most of the elastic energy may turn into kinetic energy, and the pressure gradient would change significantly during the starquake; the star would then experience a normal glitch with a sharp increase in frequency. Therefore, during a glitch, only $\boldsymbol{\omega}$ and $\boldsymbol{v}$ in equation (3) change, with equation (4) to be well approximated as $\Delta\omega/\omega \sim 10^{-9}$ and $\Delta\dot{\omega}/\dot{\omega} \sim 10^{-2}$ are very small. Then, equation (3) of the viscous fluid can be reduced to

$$\frac{\partial}{\partial t}\boldsymbol{v} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = \mu\Delta\boldsymbol{v} - (-2\,\boldsymbol{v}\times\,\omega). \quad (5)$$

We are concerned with the initial and boundary conditions for our simulations. First, we assume that only part of a solid quark star breaks under the stellar surface (the shaded region in Fig. 2), with a thickness of H. Note that this assumption is made to simplify the calculation, but it is not essential for general results in the starquake model for slow glitches (see Section 3). We use h to denote the height of matter in the broken part (i.e. $h = 0$ at the bottom while $h = H$ on the surface). Two cases are considered in the simulated results of Fig. 3, $H = 10$ per cent R and 20 per cent R, where R is the stellar radius. We suppose that the velocity of matter increases linearly from the bottom of the collapsed layer to the top, with a boundary of zero velocity at the bottom, and that the velocity increases gradually from polar to equator. The tangential velocity can then be formulated as

$$v_{\max}(\theta) = v_{\mathrm{i}}\sin\theta,$$
$$v(\theta, h) = v_{\max}(\theta)\tfrac{h}{H}, \quad (6)$$

where $\boldsymbol{v_i}$ is the maximum of the initial velocity soon after a quake. A starquake is assumed to be a trigger for the initial movement

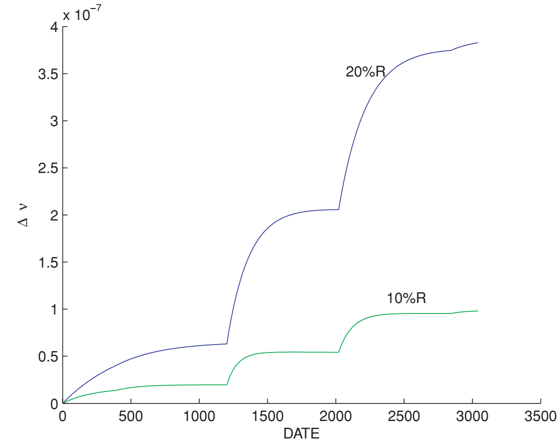

Figure 3. Simulation of slow glitches. The vertical axis shows the frequency change, $\Delta\nu$ (in units of 10^{-7} Hz), and the horizontal axis measures the time (in d). The two curvatures in this figure are two simulations, with thicknesses of the surface layer 10 per cent R and 20 per cent R, respectively. The starting point in time is at MJD 49857 in Fig. 1.

with maximum velocity v_{i}. The continuous condition of matter (i.e. $\nabla\cdot\boldsymbol{v} = 0$) determines the radial velocity, given that the viscous fluid is incompressible.

Solving equation (5) using boundary equation (6), we can obtain a three-dimensional vector field of velocity all over the star. To achieve the change of shape and rotating frequency, we adopt the incompressible hypothesis and the conservation law of angular momentum. The movement of incompressible matter in the layer gradually changes the shape of the star, and thus the angular frequency, because of the conservation of angular momentum. In fact, the conservation law of angular momentum is not obeyed precisely because of the spin-down torque. However, in light of the short duration of the glitch, the conservation can be well approximated during a relatively short time.

After a series of slow glitches, we suppose that the star remains at an equilibrium state, which is described by equation (2). It is worth clarifying that, as noted in Fig. 2 (note the difference between states 1 and 3), after each slow glitch (except the last), the star did not turn to the real equilibrium state, but actually suffered a series of glitches, not just a single glitch. We can then obtain the eccentricity of the star with the angular frequency at the end of the five slow glitches using equation (2). After this, we can calculate the star's spin frequency at any time using the conservation law of angular momentum.

From the above, we can carry out numerical calculus after setting the values of initial velocity $\boldsymbol{v_i}$ and viscosity μ. Our goal is to find whether there is reasonable parameter space (of $\boldsymbol{v_i}$ and μ) where the general features of the slow glitches observed could be reproduced. In the calculation, the 'heun' method is adopted in order to obtain more accurate values from numerical calculations.

3 RESULTS

As an example, we perform the simulation with a typical density of $\rho = 10^{14}$ g cm^{-3} and a stellar radius of $R = 10$ km. However, according to various simulations, the general conclusions would not change significantly if these two parameters were of the same order. Observational data of slow glitches and the simulated result in the solid quark star model are shown in Figs. 1(b) and 3, respectively.

Table 1. The parameters in the simulation where the thickness of the surface layer is set at 10 per cent R. The corresponding simulated result is shown in Fig. 3.

Number	μ (cm^2 s^{-1})	v_i (cm s^{-1})
1	$10^{2.4}$	$10^{-10.61}$
2	$10^{2.6}$	$10^{-10.88}$
3	$10^{2.5}$	$10^{-10.17}$
4	$10^{2.4}$	$10^{-10.00}$
5	$10^{2.6}$	$10^{-11.10}$

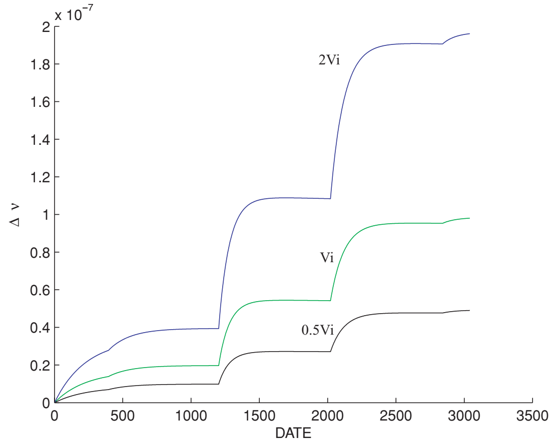

Figure 4. Different curves of $\Delta\nu$ with different parameters of v_i. The curve labelled 'Vi' is that same as that labelled '10 per cent R' in Fig. 3, with v_i values listed in Table 1. The curves labelled '2Vi' and '0.5Vi' have v_i values double and half those listed in Table 1, respectively.

The parameters μ and v_i for the five slow glitches are also shown in Table 1.

In Fig. 3, two simulated curves are shown with different thicknesses of the surface fluid layer: $H = 10$ per cent R and $H = 20$ per cent R. From Fig. 3, it is obvious that the thickness does indeed affect the result for $\Delta\nu$. Although the general behaviour remains the same, the amplitude and the gradient of $\Delta\nu$ increase as the percentage of the thickness of the stellar radius increases.

What if the values of v_i and μ in Table 1 change? For $H =$ 10 per cent R, the influences of these parameters on the simulated results of $\Delta\nu$ are shown in Figs. 4 and 5.

In Fig. 4, where values of μ are the same as those listed in Table 1, $\Delta\nu$ curves for various v_i are illustrated. It is evident that a larger value of v_i could result in a larger glitch amplitude of $\Delta\nu$. It is worth noting, however, that the time durations in which $\Delta\nu$ increases to a maximum are almost the same. This can easily be understood as a larger v_i implies a larger stress and energy released in the starquake. The effect on the amplitude of $\Delta\nu$ by changing μ values is demonstrated in Fig. 5, where the values of v_i are the same as those listed in Table 1. As shown in Fig. 5, smaller μ values can not only result in higher amplitudes of $\Delta\nu$, but also in longer durations for $\Delta\nu$ to increase to a maximum. This result can also be easily understood. In the model, the fluid moves under Coriolis force and viscous resistance. A smaller value of μ could lead to a relatively smaller resistance and thus a longer time to reach equilibrium. In summary, it is necessary to have a high value of v_i or a low value of μ in order to reproduce a large amplitude of $\Delta\nu$, while a small ν value could effectively delay the increase of $\Delta\nu$.

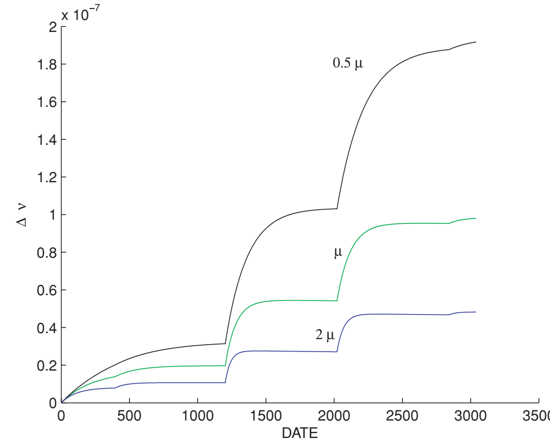

Figure 5. Different curves of $\Delta\nu$ with different parameters of μ. The curve labelled 'μ' is the same as that labelled '10 per cent R' in Fig. 3, with μ values listed in Table 1. The curves labelled '2μ' and '0.5μ' have μ values double and half those listed in Table 1, respectively.

4 CONCLUSIONS AND DISCUSSION

We suggest that a solid quark star model, in which the surface matter breaks during an initial small glitch, can be helpful for understanding the general features of slow glitches observed recently. Although this is in reality problem of rheology in principle, we deal with the matter as viscous fluid for simplicity. Numerical calculations show that the observed slow glitches can be reproduced if the effective coefficient of viscosity is $\sim 10^2$ cm^2 s^{-1} and the initial velocity of the superficial layer is of the order of 10^{-10} cm s^{-1} in the coordinate rotating frame of the star. As an example, we performed simulations with a typical density of $\rho = 10^{14}$ g cm^{-3} and a stellar radius of $R = 10$ km, but the general conclusions would not change significantly if these two parameters were of the same order.

The pieces of blocks should be recorrelated by solidifying the melted surfaces of the pieces, into which the superficial layer of a solid quark star breaks soon after a quake. The energy (both gravitational and elastic) released during a slow glitch may at first result in the melting of the conjunctional parts of the segments (i.e. the surfaces of the blocks). The temperature then increases significantly at only a small fraction ($\eta \ll 1$) of the segments. For an order-of-magnitude demonstration, we apply the Debye model to normal solid quark matter and obtain a heat capacity of

$$C_v = 0.15 \frac{k_B^4 R^3 T^3}{c^3 \hbar^3} \frac{\rho_0}{\rho_B} \eta. \tag{7}$$

The matter absorbs an energy of $E = \int_0^T C_v \mathrm{d}T$ from an initial temperature of $T_0 \ll T$ to T. For released energy of $E \sim 10^{39}$ erg and a melting temperature of $T \sim 10$ MeV ($\gg$ the star's global temperature $T_0 \sim$ keV), we have

$$\eta = 2.7 \times 10^{-22} \frac{c^3 \hbar^3}{k_B^4 R_6^3 T_{11}^4} \frac{\rho}{\rho_0} \approx 10^{-8} \frac{1}{R_6^3 T_{11}^4} \frac{\rho}{\rho_0}, \tag{8}$$

for stars with a radius of $R_6 \times 10^6$ cm and a density of ρ, where $T_{11} \times 10^{11}$ K is the melting temperature and ρ_0 is the nuclear density. For blocks with a length-scale of 1 m, this calculation shows that the heating surface part has a thickness of about 10^7 fm for each of the blocks. We note that the time-scale to maintain such a high temperature gradient should be very small, so that the contacting parts cool rapidly to solidify. More and more small blocks become correlated (i.e. a large solid-like bulk matter forms) as a slow glitch evolves, and then strain energy develops again. All the heated points are within the star and the cold surface remains, so no significant heating feature can be observed after a glitch.

Alternatively, let us consider a crusted strange quark star with a solid crust and a fluid quark matter core. The inertia of the crust can be only $\sim 10^{-5}$ times that of the quark matter core. According to $\triangle I/I \sim \triangle\Omega/\Omega \sim 10^{-8}$ for slow glitches, the crust ellipticity should change to a value of $\sim 10^{-3}$, while the actual ellipticity is $\sim 10^{-5}$ for stars with a period of $\sim$1 s. This means that pulsar slow glitches cannot be reproduced in a crusted strange star model.

What is the key ingredient that makes a pulsar undergo a normal or a slow glitch? We think that the stellar mass of a quark star could play an important role.

Let us first introduce two types of stress force inside solid stars. As noted by Xu (2006), two factors could result in the development of stress energy in a solid star, and then in star quakes as glitches. (i) As a quark star cools (even spinning constantly), the changing state of matter may cause the development of anisotropic pressure distributed inside solid matter. Such matter cannot be well approximated by a perfect fluid, and the equation governing the star's gravitational equilibrium should then not be the Tolman–Oppenheimer–Volkoff (TOV) equation. In the case of spherical symmetry (the simplest case), we can introduce the difference between radial and tangential pressures, Δ. A change of Δ would lead to no conservation of stellar volume. We call this force the bulk-variable force, which is primary in this case. This type of force may be the key factor for giant quakes during superflares of soft gamma-ray repeaters (Xu et al. 2006). (ii) A uniform fluid star would keep its eccentricity as presented in equations (1) or (2) (i.e. the eccentricity decreases as a star spins down). However, for a solid star, the shear stress would prevent the eccentricity of the star from decreasing during spin-down. In this case, even the state of matter does not change, and stress energy could still develop as the solid star spins down. We call this type of force the bulk-invariable force, as the total stellar volume may always remain constant. Starquakes that are a result of bulk-invariable force, and the consequent glitches, have been calculated and discussed in Zhou et al. (2004).

Both bulk-invariable and bulk-variable forces can result in decreases of moment of inertia, and therefore in pulsar glitches. These two types of force could trigger normal glitches if they were relatively stronger than the critical stress, but might only produce slow glitches if weaker. The bulk-variable force could be stronger than the bulk-invariable force for massive solid stars, because, in a special case of non-rotation, the gravity $\propto M^2/R^2$ (where M is the mass and R is the radius) is strong there. If the critical stress of solid quark matter is almost the same (note that the effective critical stress could be much small for a shiver-like surface layer), we think that normal glitches are more likely to occur in massive quark stars with possibly strong bulk-variable forces, while slow glitches are more likely to take place in low-mass solid quark stars.

What if PSR B1822–09 is a low-mass quark star? The spin-down as a result of the magnetodipole torque for a star with a magnetic dipole moment μ, a moment of inertia I and an angular velocity Ω is

$$\dot{\Omega} = -\frac{2}{3Ic^3}\mu^2\Omega^3. \tag{9}$$

This rule remains quantitative for any oblique rotators, as long as the braking torques as a result of magnetodipole radiation and the unipolar generator are combined (Xu & Qiao 2001). Approximating $I = (2/5)MR^2$, $\mu = (1/2)BR^3$ and $M = (4\pi/3)R^3\rho$, with ρ being the average density, we then have

$$R = 2.76 \times 10^{31}\rho_{14}B^{-2}\ \mathrm{cm}. \tag{10}$$

where $\rho = \rho_{14} \times 10^{14}$ g cm^{-3}, and $P = 0.769$ s and $\dot{P} = 5.23 \times 10^{-14}$ for PSR B1822–09. The pulsar would be a few km in radius if the polar magnetic field were $\sim 10^{13}$ G.

The potential drop in the open field-line region, ϕ, should be greater than the critical value of $\sim 10^{12}$ V $\simeq 3 \times 10^9$ e.s.u. in order to create secondary $e^{\pm}$ plasma via gap discharges. The potential drop between the centre and the edge of the polar cap is therefore (Ruderman & Sutherland 1975)

$$\phi = \frac{2\pi^2}{c^2}R^3BP^{-2} = 7.84 \times 10^{74}\rho_{14}^3B^{-5}. \tag{11}$$

Note that the potential in the above equation would be higher if the effect of inclination angle were included (Yue, Cui & Xu 2006). The drop could be high enough for pair production if the field $B \sim 10^{13}$ G, and the pulsar should then be radio loud.

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