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Colour-Charged Quark Matter in Astrophysics? *

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Colour confinement is only a supposition, which has not yet been proven in QCD. Here we propose that macroscopic quark–gluon plasma in astrophysics could hardly maintain colourless because of causality. It is expected that the existence of chromatic strange quark stars as well as chromatic strangelets preserved from the QCD phase transition in the early Universe could be unavoidable if their colourless correspondents do exist.

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The elementary strong interaction is believed to be recognized by two distinct features: asymptotic freedom and colour confinement. Though Politzer[1] and Gross and Wilczek[2] proved asymptotic freedom in quantum chromodynamics (QCD), QCD in nonperturbative regime is still unsolved and becomes one of the top challenges for physics today. Nambu[3] discussed the possibility of colour confinement as the QCD vacuum is supposed to be a condensation of virtual quark–antiquark pairs and gluons. In the strong coupling approximation, lattice gauge calculation shows that the QCD potential is linear in the infrared region.[4] Certainly, we have never found a chromatic particle in an accelerator experiment. However, unfortunately, all the evidence mentioned above should not be enough to convince us of a strictly held nature of colour confinement, since infrared and ultraviolet regions could be separated by one or more discontinuous phase transitions.[4] It is still not sure whether or not the de-confinement and chiral restoration coincide at high energy scale.[5]

The special QCD vacuum may probably explain the colour–singlet of particles in the high energy experiments. Colour-charged quarks, exchanging gluons, are supposed to be confined in colour-neutral hadrons (baryons or mesons) with others. When one of the quarks in a given particle is ‘pulled’ away from its neighbours, it would be energetically favourable for the virtual quark–antiquark pairs to become valency and to keep the hadrons in singlet states again. However, things would be much different in the case of a huge bulk of astrophysical quark–gluon plasma. Let us consider a bulk of quark matter which is initially colourless. It should evaporate or split if it has to reduce its volume for certain reasons. In the case of evaporating particles (e.g. baryons or mesons), colour–singlet might preserve easily since this evaporation could be considered as a *local* process (note that QCD is a local gauge theory, and colour confinement would then be a local concept). However, in the splitting process, it could be difficult for us to believe that every splitting piece *occurs* to be colourless because the time needed for transforming information (even as fast as light) from one part of the bulk to the other should not be negligible in this non-local case (Fig. 1).

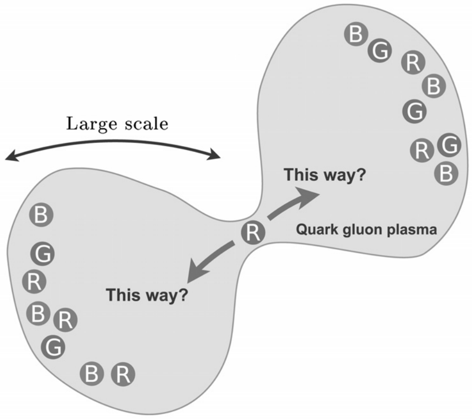

Fig. 1. An illustration of splitting a large bulk of quark matter. If a red quark in the centre (labelled as R) wants to decide which way to go, it should be able to count *instantaneously* the colour–charges of quarks in at least one side to keep the divided two parts in colour–singlet states. It could then be very difficult to maintain colour–singlets of the splitting pieces in a macroscopic scale.

There is a great difference between electroneutral and colourless states. An electrically charged body may discharge via ejecting electrons or operating virtual $e^{\pm}$ pairs in vacuum. How about chromatic quark matter? Note that the QED vacuum is screening, while the QCD vacuum is anti-screening. It might be energetically favourable for chromatic quark matter to locally excite and eject colour–singlet particles, and the colour–charge is then maintained. The interaction between chromatic bulks of quark matter might only be in a very short distance *if* single colour–gluon interchange is not allowed since mesons or glueballs as intermediate particles are very massive. All the observations mentioned above could show us that it would be hard for chromatic quark matter to drop its colour off unless by random collision.

If colour confinement is not *exactly* held, chro-

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matic quark matter could be in an extremely high energy scale, which should be too high for accelerators to achieve nowadays. Surely, the energy scale would be hard to estimate in the framework of quantum field theory since the matter is in a non-local and bound state. We have tried two phenomenological models[6] to make sense of this. In the first model, we considered a bulb in a colour dielectric medium[7] which has colour dielectric constant less than one. The bulb can change its radius to minimize its total energy. In the second model, we simply laid up a lot of colour dipoles in the vacuum and calculate their respondence for a valance colour charge in the centre. Both models show us that the minimum energy scale $U_{\min}$ for exciting chromatic quark matter should depend on the radius R of the bulk, with a likely relation of $U_{\min} \simeq \text{const} \cdot 1/R$. The constant in this relation can be estimated if we pay attention to the fact that no accelerator (which has an energy scale of about 100 GeV) is powerful enough to create a chromatic particle (which has a typical radius of about 1 fm). The constant could then be greater than $\sim 10^{-10}$ MeV·m. It is observed then that colour confinement can only be hold exactly only if the constant is infinity. Therefore, the conclusion could be that, if colour confinement is exactly hold for microscopic particles ($\sim$ 1 fm), it is also exactly hold for a bulk of quark matter; but if it is just approximately hold, to create a bulk of chromatic quark matter is much easier (i.e. to need lower energy) than to create a chromatic microscopic particle.

The first implication of chromatic quark matter could be of splitting strange (quark) stars. Strange stars are potential candidates for the nature of pulsar-like stars (Refs. [8,9], and see Ref. [10] for a review). They are a kind of fermion stars which are bound by strong interaction (and gravity if stellar masses approach the maximum limit) and are in fact large bulks of quark–gluon plasma. There could be two conceivable channels to split bulk quark matter: merge of binary strange stars and supernova explosion. It is very uncertain to calculate the former process, and we then think about the latter one only as follows.

Based on the kick velocity $v_{\rm k}$ of pulsars, in which we choose 1000 km·s^{-1} as an upper limit,[11] the kinematic energy scale of supernova explosion can be estimated. Assuming an equipartition rule for stellar rotational energy and kick energy, we could then estimate the critical condition for splitting a strange star.[6] The relation between spin frequency and equilibrium shape of a liquid star was given by Maclaurin in the framework of Newtonian gravity, with an equation[12] of

$$\Omega^2 = 2\pi G\rho\Big[\frac{\sqrt{1-e^2}}{e^3}(3-2e^2)\arcsin e - \frac{3(1-e^2)}{e^2}\Big], \tag{1}$$

where ρ is the density of the star, Ω is the angular velocity, e is the eccentricity of the meridian plane, and G is the Newtonian gravitational constant. A rotating star would disintegrate at the bifurcation point of the Jacobian sequence branches, with $e = 0.8127$.[13,14] Assuming $I\Omega^2 = Mv_{\rm k}^2$, one can calculate the rotational energies for different stellar masses M, where I is the momentum of inertia and $v_{\rm k} = 10^8$ cm/s. In Fig. 2, the rotation energy $E_{\rm r,split}$ of a star with $e = 0.8127$ is shown by the solid line (i.e. e in Eq. (1) is fixed), whereas the actual rotation energy is shown by the dashed line (i.e., Ω is determined by $v_{\rm k}$). Comparing those two kinds of rotation energies, we can see that a strange quark star with initial matter of $\sim 10^{-5} M_\odot$ may split into two or more bulks of quark matter.

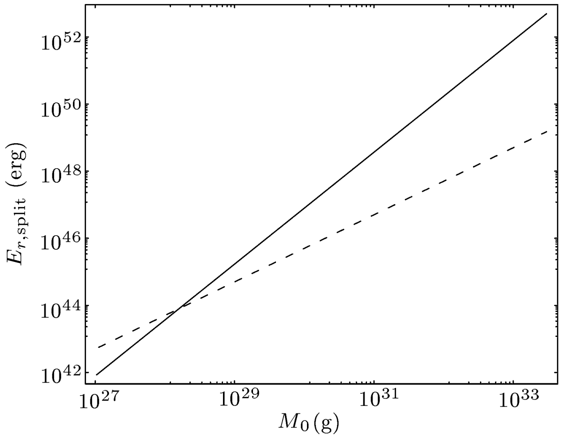

Fig. 2. Comparison of the rotation energy that is needed for a star to split (solid) and the rotation energy that the star may actually have (dashed line) for varying stellar mass (M_0). In the calculation, we choose $\rho = 4.1 \times 10^{14}$ g·cm^{-3} for low mass strange stars. It is shown that a strange quark star as low as $10^{-6} M_\odot$ could split.

The calculation shown in Fig. 2 is only applicable to rigidly rotating bodies kicked during the supernova explosions, but actually a star may not be in this situation. During the birth of protoquark stars with high temperature T and high ∇T,[15] hot turbulently convective quark stars may eject low-mass quark matter (quark nuggets).[16] Also during supernova explosions, small quark nuggets could possibly be kicked out from the protoquark stars if they are not rigid rotators. For the sake of simplicity, we assume that the energy kicked to the small piece is the same as that to the quark star (which is related to the observable kick velocity of pulsars). To compensate for the gravitational potential energy (for a quark star not too massive, the typical density is a constant) in the framework of Newtonian, the maximum piece can be kicked to infinity has a typical mass $M = 7.5 \times 10^{28}$ g and radius $R = 0.35$ km (for a constant density of $\rho = 4.1 \times 10^{14}$ g·cm^{-3}). In the above calculation we neglect the energy loss caused by gravitational radiation, which is very effective[6] but difficult to calculate. The maximum mass of splitting pieces might then be actually smaller than $\sim 10^{28}$ g. This kind of splitting pieces of quark matter could be candidates for the planets of pulsars.[17,18] However, it is worth noting that, even though the kicked nuggets take colour-charges, the colours might not have dynamical contribution to be as strong as gravity if chromatic quark matter would not interact at a long distance. Anyway, a detail investigation of the extra colour interaction in chromatic quark-star and quark-planet system could be helpful to obtain observational evidence for quark matter with colour charges.

The second implication could be about the chromatic strangelets preserved during the QCD phase transition of the early Universe. In the standard model,[19] a first-order QCD transition leads to bubble nucleation. Bubbles of quark-gluon plasma form during the transition. In the case of homogeneous nucleation, the mean distance of nucleation, $d_{\rm nuc}$, could be $< 2\,{\rm cm}$, while the value of $d_{\rm nuc}$ may be several meters in the case of heterogeneous nucleation.[20] The strangelets proposed by Witten[21] seems to be impossible since the suggestion needs $d_{\rm nuc} \gtrsim 300\,{\rm m}$.[19] However, things would be much different if chromatic strangelets were created during the cosmic QCD phase-transition. As we have mentioned, it could be energetically favourable to exist huge bulk of chromatic quark matter since $U_{\rm min} \propto R^{-1}$. In this sense, the hadronization of chromatic quark nuggets could not be very effective and may still be residual today.

Let us estimate roughly the number density of such strangelets in the Universe, assuming that all the strangelets take colour-charges and could survive. Lattice gauge calculations show us that the QCD phase transition occurs at a temperature of $T_{\rm c} = 170\,{\rm MeV}$.[19] Using the relation of $a(t)T(t) = {\rm const}$ for radiation field in the expanding Universe, where the scale factor $a(t)$ and the temperature $T(t)$ are as functions of cosmic time t, and choosing $d_{\rm nuc} = 2\,{\rm cm}$ and $T({\rm today}) = 2.73\,{\rm K}$ of cosmic microwave background, we obtain a number density of about $(0.1\,{\rm AU})^{-3}$ in today's Universe.

It is not easy to estimate the mass of that kind of strangelets since one can not have a believable method to calculate the total energy of chromatic quark matter. A possible restriction for the mass could be obtained by noting that the total mass of these strangelets should be lower than that of dark matter. If we choose the Hubble constant $H_0 = 72\,{\rm km}\cdot{\rm s}^{-1}\cdot{\rm Mpc}^{-1}$ and assume that 25% of the total energy density of the Universe is in the form of dark matter in the concordance model,[19] we could have an upper limit of the strangelet mass: $m_{\rm s} \lesssim 7.4 \times 10^6\,{\rm g}$. The radius of strangelets with mass of $\sim 10^6\,{\rm g}$ is about $10^{-3}\,{\rm cm}$. It is possible that such strangelets, which are lighter than asteroids, are wandering in our solar system, and we can hardly observe them because they seems to contribute only gravitational interaction in the space. Nevertheless, these strangelet bummers may have had significant consequence during the early Universe (e.g. the period of galaxy formation).

Is there any experimental feature of chromatic quark matter? This is really an interesting question to be answered by more investigations in the future. Similar to the Schwinger process[22] of the QED vacuum polarization by a prescribed electromagnetic field, colour-charged quark matter may try to radiate colour-charged anti-particles but it is forbidden by colour-confinement or approximate colour-confinement since small colour-charged particles should have high energy barrier as we show above. Coloured particles will transfer kinematic energy to colourless ones (maybe by electromagnetic force) and may radiate colourless hadrons in the nearby polarized QCD vacuum, but could become more and more difficult and may finally stop when its mass decreases to a critical value due to the fact of $U_{\rm min} \propto 1/R$. A particularly fascinating process could be that a chromatic relativistic strangelet goes into Earth's atmosphere. It may absorb nucleons first, and then its temperature increases to be high enough to evaporate hadrons, and may finally become a particle in the Earth, keeping its initial colour. The character of its atmospheric shower depends on the detail interactions, which is certainly model-dependent. Are some cosmic events (e.g. the ultra-high energy cosmic rays) related really to chromatic strangelets?[18] We cannot know at this time.

Let us summarize briefly our opinions. Chromatic strange quark stars as well as strangelets could be unavoidable if their colourless correspondents do exist, but evidence for colour-charges might hardly be obtained nowadays. We suggest that colour confinement could not be held exactly in the nature.

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