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The formation of submillisecond pulsars and the possibility of detection

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ABSTRACT

Pulsars have been recognized to be normal neutron stars, but sometimes have been argued to be quark stars. Submillisecond pulsars, if detected, would play an essential and important role in distinguishing quark stars from neutron stars. We focus on the formation of such submillisecond pulsars in this paper. A new approach to the formation of a submillisecond pulsar (quark star) by means of the accretion-induced collapse (AIC) of a white dwarf is investigated. Under this AIC process, we found that: (i) almost all newborn quark stars could have an initial spin period of ∼0.1 ms; (ii) nascent quark stars (even with a low mass) have a sufficiently high spin-down luminosity and satisfy the conditions for pair production and sparking process and appear as submillisecond radio pulsars; (iii) in most cases, the times of newborn quark stars in the phase with spin period <1 (or <0.5) ms are long enough for the stars to be detected.

As a comparison, an accretion spin-up process (for both neutron and quark stars) is also investigated. It is found that quark stars formed through the AIC process can have shorter periods (≤0.5 ms), whereas the periods of neutron stars formed in accretion spin-up processes must be longer than 0.5 ms. Thus, if a pulsar with a period shorter than 0.5 ms is identified in the future, it could be a quark star.

Key words: accretion, accretion discs – gravitational waves – stars: Neutron – pulsars: general.

1 INTRODUCTION

Although it has been more than 40 years since the discovery of radio pulsars, their real nature is still not clear, owing to a lack of knowledge about cold matter at supranuclear densities. Neutron matter and quark matter are two conjectured states for such compact objects. Objects consisting of the former are called neutron stars, and those consisting of the latter are called quark stars. It is an astrophysical challenge to distinguish real quark stars from neutron stars observationally (see reviews by, for example, Madsen 1999; Glendenning 2000; Lattimer & Prakash 2001; Kapoor & Shukre 2001; Weber 2005; Xu 2008). The most obvious discrepancy could be the minimal spin period of these two distinct kinds of objects. The minimal periods of these two kinds of objects are related to their formation process. How fast a neutron star or a quark star can rotate during the recycling process in low-mass X-ray binaries (LMXBs) has been considered by a number of authors (Bulik, Gondek-Rosińska & Kluźniak 1999; Blaschke et al. 2002; Zdunik, Haensel & Gourgoulhon 2002; Xu 2005; Arras 2005). Friedman, Parker & Ipser (1984) found that neutron stars with the softest equation of state can have a spin period as short as 0.4 ms. The shortest spin period for neutron stars computed by Cook, Shapiro & Teukolsky (1994) is about 0.6 ms. Frieman & Olinto (1989) showed that the maximum rotation rate of secularly stable quark stars may be less than 0.5 ms. Burderi & D'Amico (1997) discussed a possible evolutionary scenario resulting in a submillisecond pulsar, and the possibility of detecting a submillisecond pulsar with a fine-tuned pulsar-search survey. Gourgoulhon et al. (1999) investigated the maximally rotating configurations of quark stars and showed the minimal spin period to be between 0.513 and 0.640 ms. Burderi et al. (1999) predicted that there might exist an as-yet undetected population of massive submillisecond neutron stars, and the discovery of a submillisecond neutron star would imply a lower limit for its mass of about $1.7\,M_{\odot}$. A detailed investigation of the spin up of neutron stars to submillisecond periods, including a complete statistical analysis of the ratio with respect to normal millisecond pulsars, was performed by Possenti et al. (1999). The minimal recycled period was found to be 0.7 ms. Gondek-Rosińska et al. (2001) found that the shortest spin period is approximately 0.6 ms through the maximum orbital frequency of accreting quark stars. Huang & Wu (2003) found the initial periods of pulsars to be in the range of $0.6 \sim 2.6$ ms using proper motion data. Zheng et al. (2006) showed that hybrid stars, instead of neutron or quark stars, may lead to submillisecond pulsars. Haensel, Zdunik & Bejger (2008) discussed the equation of state (EOS) of compact stars and the spin up to a

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submillisecond period by means of mass accretion from a disc in a LMXB.

There have been many observational attempts to find submillisecond pulsars. A possible discovery of a 0.5-ms pulsar in Supernova 1987A was not verified in follow-up observations (Sasseen 1990; Percival et al. 1995). Bell et al. (1995) reported on optical observations of the low-mass binary millisecond pulsar system PSR J0034-0534 and used white dwarf cooling models to speculate that the limit magnitude of J0034-0534's companion suggested that the initial spin period of this millisecond pulsar was as short as 0.6 ms. As addressed by D'Amico & Burderi (1999), the detection of a pulsar with a spin period well below 1 ms could put severe constraints on the neutron star structure and the absolute ground state for baryon matter in nature. They designed an experiment to find submillisecond pulsars with the Italian Northern Cross radio telescope near Bologna. Edwards, van Straten & Bailes (2001) found no submillisecond pulsars in a search of 19 globular clusters using the Parks 64-m radio telescope at 660 MHz with a time resolution of 25.6 μs. Han et al. (2004) did not find any submillisecond pulsars in the highly polarized radio source of the NVSS (NRAO VLA Sky Survey). Kaaret et al. (2007) found oscillations at a frequency of 1122 Hz in an X-ray burst from the transient source XTE J1739-285, which may contain the most rapidly rotating neutron star discovered to date. Significant difficulties do exist for the detection of binary submillisecond pulsars in current radio surveys as a result of strong Doppler modulation and computational limitations (Burderi et al. 2001).

How do submillisecond pulsars form? This is an open question, and one that we will explore in this paper. Previously, discussions concentrated on the formation of neutron stars or quark stars through a process of spin up by means of accretion in binaries. We consider a new approach to creating a submillisecond pulsar (quark star) with super-Keplerian spin through the accretion-induced collapse (AIC) of a massive white dwarf (WD). The initial spin of the newborn quark star could be super-Keplerian, and it could have a long lifetime in the submillisecond phase and produce enough strong radio emission to be detected.

In Section 2 we discuss low-mass quark stars formed from the AIC of WDs, which can have a minimal initial period of submilliseconds. In Section 3, the radiation parameters and the conditions for pair production are estimated in order to investigate whether the AIC-induced quark stars could be pulsars or not. The lifetimes of submillisecond pulsars are also estimated, and the possibility of detection is discussed. The spin-down evolution diagrams of a newborn quark star and neutron star are also plotted. In Section 4, as a comparison, the submillisecond pulsars formed through accretion acceleration (spin up) in binary systems are considered. In Section 5, conclusions and discussions are presented.

2 SUBMILLISECOND QUARK STARS FORMED THROUGH THE AIC OF WDS

The formation of a neutron star from the AIC of a massive WD has been discussed by many authors (Nomoto et al. 1979; Nomoto & Kondo 1991; van Paradijs et al. 1997; Fryer et al. 1999; Bravo & García-Senz 1999; Dessart et al. 2006). Recently, it has been pointed out that the Galactic core-collapse supernova rate cannot sustain all the neutron star populations (Keane & Kramer 2008), which implies that other mechanisms for forming neutron stars must exist. The AIC of a massive WD can be an important mechanism for pulsar formation, even for isolated pulsars, if the binary systems are destroyed by strong kicks. We now discuss the possibility of the formation of a low-mass quark star through the AIC of a WD. In a binary system, when the WD has accreted enough matter from its companion that its mass reaches the Chandrasekhar limit, the process of electron capture may induce gravitational collapse. The detonation waves burn nuclear matter into strange quark matter, which spreads out from the inner core of the WD (Lugones, Benvenuto & Vucetich 1994). A boundary of strange quark matter and nuclear matter will be found at the radius at which the detonation waves stop, when the nuclear matter density drops below a critical value. A similar process was discussed and calculated by Chen, Yu & Xu (2007). The size of the inner collapsed core may depend on the chemical composition and accretion history of the WD (Nomoto & Kondo 1991). Consequently, quark stars with different masses could be formed.

Both rigidly and differentially rotating WDs are considered. As a first step, for simplicity, we assume that both the collapsed WD and the newborn quark star have rigidly rotating configurations. The WDs, progenitors of these quark stars, could have a uniformly rotating configuration owing to the effects of crystallization; in addition, an increase in the central density may lead to catastrophic evolution (supernova) (Koester 1974). With these assumptions, a model of the formation of submillisecond pulsars is given below. The initial spin period of AIC-produced quark stars can be estimated as follows. We assume that the mass ($M_\star$) of the nascent quark star ranges from $10^{-3}\,\mathrm{M}_\odot$ to $1\,\mathrm{M}_\odot$, and that the WD rotates rigidly at the Kepler period (P_K) just before collapse. The rest mass of the quark star ($M_\star$) is approximately equal to the mass (m_core) of the inner collapsed core of the WD. If the angular momentum is conserved during AIC, the newborn quark star can rotate at a much shorter period, P_q, and then

$$I_\mathrm{core}\frac{2\pi}{P_K} = I_\mathrm{q}\frac{2\pi}{P_\mathrm{q}}. \tag{1}$$

That is to say,

$$P_\mathrm{q} = \frac{I_\mathrm{q}}{I_\mathrm{core}}P_\mathrm{K}, \tag{2}$$

where I_q is the moment of inertia of the quark star, and I_core is the moment of inertia of the inner collapsed core of the WD, which can be well approximated by

$$I_\mathrm{core} \simeq \frac{2}{5}M_\mathrm{core}R_\mathrm{core}^2. \tag{3}$$

The mass and radius of a low-mass ($M_\star \leqslant 1\,\mathrm{M}_\odot$) quark star could be approximately related by $M_\star = (4/3)\pi(4\beta)R^3$ (Alcock, Farhi & Olinto 1986) in the bag model. We have an approximate formula for the moment of inertia of the fast-rotating quark star:

$$\begin{aligned} I_\mathrm{q} &= 2\int_0^R dz\int_0^{\sqrt{R^2-z^2}}\frac{4\beta 2\pi x^3}{\sqrt{1-\frac{4\pi^2x^2}{c^2P_\mathrm{q}^2}}}dx \\ &= \frac{\beta c P_\mathrm{q}}{16\pi^4}\Bigg[6\pi c^3P_\mathrm{q}^3R - 8\pi^3cP_\mathrm{q}R^3 + \Big(16\pi^4R^4 \\ &\quad + 8\pi^2c^2P_\mathrm{q}^2R^2 - 3c^4P_\mathrm{q}^4\Big)\ln\frac{1+\frac{2\pi R}{cP_\mathrm{q}}}{\sqrt{1-\frac{4\pi^2R^2}{c^2P_\mathrm{q}^2}}}\Bigg], \end{aligned} \tag{4}$$

where the z-axis is the spin axis; x is an integral variable of each disc perpendicular to the spin axis; c is the speed of light; R is the radius of the quark star; and the bag constant β of quark stars is 60–110 MeV fm^{-3}, that is, (1.07–1.96) $\times\ 10^{14}$ g cm^{-3}, β_{14} in units of 10^{14} g cm^{-3}.

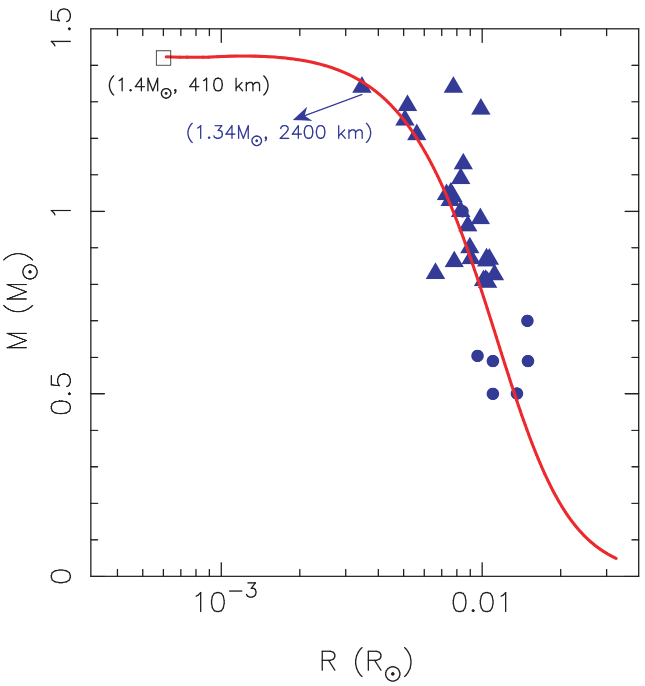

Figure 1. The relationship between mass and radius for white dwarfs (WDs). The red line is the theoretical line. The blue triangles and circles are observed WD data, taken from table 1 of Należyty & Madej (2004) and tables 3 and 5 of Provencal et al. (1998), respectively. Among these data, the WD RE J0317-853 is the most massive WD, with mass and radius $1.34\,\mathrm{M_\odot}$ and 2400 km respectively. The square represents a WD with the Chandrasekhar mass limit.

For the WD, using both the non-relativistic hydrostatic equilibrium equation

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{Gm(r)\rho(r)}{r^2}, \tag{5}$$

and the general EOS for a completely degenerate Fermi gas,

$$\begin{aligned} p &= \frac{1}{3\pi^2\hbar^3}\int_0^{p_\mathrm{F}} \frac{c^2p^4}{\sqrt{c^2p^2+m^2c^4}}\mathrm{d}p \\ &= 1.42\times10^{25}\phi(x)\ \mathrm{dyn\ cm^{-2}}, \end{aligned} \tag{6}$$

where $x \equiv p_\mathrm{F}/mc, \lambda_\mathrm{e} = \hbar/(mc)$ is the electron's Compton wavelength, P_F is the Fermi momentum and

$$\begin{aligned} \phi(x) = (8\pi^2)^{-1}\Big\{&x(1+x^2)^{1/2}(2x^2/3-1) \\ &+ \ln[x+(1+x^2)^{1/2}]\Big\}, \end{aligned}$$

we calculate the mass (m_core) and the moment of inertia (I_core) of the collapsed core of a massive WD, where p, ρ, G and $\hbar$ are the pressure, the mass density, the gravitational constant and the Planck constant, respectively.

Using equations (5) and (6), one can undertake a numerical calculation to obtain the theoretical relationship between the mass and radius of the WD (red line in Fig. 1). For comparison, a similar figure is presented on the web page.[1] Before it collapses, the mass of a WD is close to the Chandrasekhar mass limit, as high as $M_\mathrm{WD} = 1.4\,\mathrm{M_\odot}$ (Shu 1982), and the corresponding radius, $R_\mathrm{WD} = 410$ km, is much smaller than those of typical observed WDs. If the initial periods, P_q, of nascent quark stars with different masses are obtained numerically from equation (2), it is found that almost all of the values of P_q are around ∼0.1 ms (see Table 1) if the WD rotates rigidly at an almost Kepler period because of accretion (or spin up) in a binary just before collapse. The surface spin velocities of the newborn quark stars are well above the Kepler velocities: we term this the 'super-Keplerian' case.

WDs may be rotating differentially, and the detailed calculations for this case are given in Appendix A. As a follow-up second step, we therefore also use equations (2), (5), (6) and (A2) to calculate the initial spin period of the nascent quark stars in the differentially rotating WD model, taking the free parameter $a = 0.5$. The results are shown in Table 1.

A newborn quark star could certainly rotate differentially, and may relax to become a rigidly rotating configuration finally. However, the time-scale of the relaxation depends on the viscosity and on the state of the cold quark matter (Xu 2009). The newborn quark star's relaxation (from a differentially rotating configuration to a rigidly rotating configuration) may be caused by rapid solidification after birth. A calculation shows that the solidification time-scale is only 10^3–10^6 s (Xu & Liang 2009). Therefore, the relaxation time-scale could be much shorter than the lifetime of pulsars with submillisecond periods (see Section 3.3 below).

The WD RE J0317-853 has the highest observed mass ($1.34\,\mathrm{M_\odot}$, close to the Chandrasekhar limit), with a radius of 2400 km (Należyty & Madej 2004). If a WD like RE J0317-853 were in a binary and accreted enough material to reach the Chandrasekhar limit, it may collapse. We therefore calculated the initial spin periods $\widehat{P}_\mathrm{q}$ and $\widehat{P}_\mathrm{dif}$ of a nascent quark star under this assumption. The calculated results are listed in Table 1. It is found that, even if a WD has a larger radius, such as 2400 km, it can also collapse to a submillisecond quark star for both rigidly and differentially rotating WD models. In the differentially rotating WD model, it tends to give a rigidly rotating configuration in the limit of large values of a: P_dif increases as the parameter a increases. The conclusions from the rigid-rotation model are valid even if differential rotation is included.

Can a quark star survive even if it rotates with such a high frequency (∼10^4 Hz)? Will it be torn apart by the centrifugal force? There are a number of features that distinguish neutron and quark stars. A low-mass quark star can spin at a super-Keplerian frequency because it is self-bound by the strong interaction. On one hand, as noted by Qiu & Xu (2006), astrophysical quark matter splitting could be colour-charged if colour confinement cannot be held exactly because of causality. On the other hand, however, rapidly spinning quark matter is unlikely to split if colour confinement is held exactly. In addition, the recently discovered nature of strongly coupled quark gluon plasma (sQGP) as realized at the Relativistic Heavy Ion Collider (RHIC) experiment (e.g. Shuryak 2006) may also prevent a super-Keplerian quark star from splitting.

The short spin period above is not surprising, and could be verified for a simplified special case, if both the density of the quark star ($=4\beta$) and the density of the WD ($=\rho_\mathrm{c}$) are uniform. Using equation (2) and the mass–radius relationship, we find the initial period of the quark star to be $P_\mathrm{q} = (\rho_\mathrm{c}/4\beta)^{2/3}\ P_\mathrm{WD} \sim 4\times10^{-3}\ (\rho_{11}/\beta_{14})^{2/3}P_\mathrm{WD}$ (with P_WD the spin period of the WD, $\rho_{11} = \rho_\mathrm{c}/10^{11}\,\mathrm{g\,cm^3}, \beta_{14} = \beta/10^{14}\,\mathrm{g\,cm^3}$), which depends only on the densities of the WD and the quark star.

If the WD has not been spun up fully to the Kepler period; that is, if the WD rotates at a sub-Keplerian period (e.g. several times P_K) before AIC, can the initial period of a newborn quark star formed

[1] http://cococubed.asu.edu/code_pages/coldwd.shtml

Table 1. The minimal initial period (P_q) and lifetimes (τ) considering the gravitational wave and electromagnetic radiation in the submillisecond-period phase for quark stars with various masses in the super-Keplerian case. P_q and P_{dif} are calculated from angular momentum conservation using a rigidly and differentially rotating white dwarf (WD) model with a central density of 10^{11} g cm^{-3}. $\widehat{P}_q$ and $\widehat{P}_{dif}$ are similarly calculated, but using a WD like RE J0317-853 with a mass of 1.4 M$_\odot$ and a radius of 2400 km. τ_1 is the time the quark star spends in the phase with period <1 ms, whereas τ_2 is the time spent in the phase with period <0.5 ms. P_q is also used in Tables 2 and 3 and Fig. 2. β is the bag constant and ε_e is the gravitational ellipticity.

Mass (M$_\odot$)	Radius (km) $\beta = 60$ MeV fm^{-3}	P_q(ms) $\beta = 60$ MeV fm^{-3}	P_{dif}(ms) $a = 0.5$	$\widehat{P}_q$(ms) $\beta = 60$ MeV fm^{-3}	$\widehat{P}_{dif}$(ms) $a = 0.5$
0.001	1.04	0.0699	0.0261	0.0481	0.0252
0.01	2.24	0.0751	0.0472	0.0512	0.0470
0.1	4.81	0.104	0.101	0.102	0.101
1	10.37	0.221	0.218	0.218	0.218

Mass(M$_\odot$)	P_q(ms)	τ_1(yr)		τ_2(yr)	
		$\varepsilon_e = 10^{-6}$	$\varepsilon_e = 10^{-9}$	$\varepsilon_e = 10^{-6}$	$\varepsilon_e = 10^{-9}$
0.001	0.0699	3.4×10^7	4.5×10^{10}	2.1×10^6	1.1×10^{10}
0.01	0.0751	7.3×10^5	2.0×10^{10}	4.5×10^4	4.3×10^9
0.1	0.104	1.6×10^4	5.4×10^9	9.0×10^2	6.5×10^8
1	0.221	3.4×10^2	3.1×10^8	2.2×10^1	2.0×10^7

from such a WD be submillisecond? We investigated the case of a massive WD (1.4 M$_\odot$, 410 km) rotating at a period $P_{WD} = 5P_K \sim$ 600 ms. The initial spin period of quark stars with various masses are as follows: $\hat{P}_q \sim 0.11$ ms for a quark star with a mass of 0.001 M$_\odot$; $\sim$0.24 ms for 0.01 M$_\odot$; 0.35 ms for 0.1 M$_\odot$ and $\sim$0.36 ms for 1 M$_\odot$. The spin-down feature of such a newborn quark star depends on its gravitational wave radiation and magnetodipole radiation (for details see Section 3).

3 RADIATION OF SUBMILLISECOND QUARK STARS WITH LOW MASSES

The mass of most submillisecond quark stars formed from the AIC of WDs is so low that we need to ask whether the quark star can produce radiation luminous enough to be observed as millisecond pulsars. The answer is related to two factors. First of all, is the rotational-energy-loss rate high enough to power the electromagnetic radiation as normal pulsars? Second, is the potential drop in the inner gap high enough for pair production and sparking to take place in the inner gap? These are necessary conditions for radio emission from pulsars.

3.1 The spin-down power of submillisecond pulsars

Normal radio pulsars are rotation-powered, and the radiation energy comes from the rotational energy loss. Here we first neglect gravitational wave radiation, and thus the rate, $\dot{E}_{rot}$, is

$$\dot{E}_{rot} = \frac{8\pi^4 R^6 B^2 P^{-4}}{3c^3}. \tag{7}$$

Comparing the rotational-energy-loss rate ($\dot{E}_{rot,q}$) of quark stars with that of normal neutron stars ($\dot{E}_{rot,NS}$), one has

$$\dot{E}_{rot,q}/\dot{E}_{rot,NS} = \frac{R_q^6 B_q^2 P_q^{-4}}{R_{NS}^6 B_{NS}^2 P_{NS}^{-4}}. \tag{8}$$

If we take normal parameters, such as the surface magnetic field of the polar cap $B_q = 10^8$ G, $B_{NS} = 10^{12}$ G, the rotational period $P_q = 0.1$ ms and $P_{NS} = 1$ s, the result is $\dot{E}_{rot,q}/\dot{E}_{rot,NS} = 10^2$ even for a quark star with a mass of 0.001 M$_\odot$. This means that the quark stars have enough rotational energy to radiate, at hundred times the rate of normal pulsars, even when the mass is so low.

3.2 Particle acceleration for submillisecond pulsars

In most radio-emission models of pulsars, such as the RS model (Ruderman & Sutherland 1975, hereafter RS75), the inverse Compton scattering (ICS) model (Qiao & Lin 1998), the multi-ring sparking model (Gil & Sendyk 2000), the annular gap model (Qiao et al. 2004) and so on, the potential drop in the inner gap must be high enough that the pair production condition can be satisfied.

In the inner vacuum gap model, there is a strong electric field parallel to the magnetic field lines resulting from the homopolar generator effect. The particles produced through the $\gamma - B$ process in the gap can be accelerated to ultra-relativistic energies (i.e. the Lorentz factor can be 10^6 for normal pulsars). The potential across the gap is (RS75)

$$\triangle V = \frac{\Omega B}{c} h^2, \tag{9}$$

where Ω is the angular frequency of the pulsar; h is the gap height; and B and c represent the magnetic field at the surface of the neutron star and the speed of light, respectively. As h increases and approaches the radius of the polar cap, r_p, the potential drop along a field line traversing the gap, cannot be expressed by equation (9) above. In this case the potential can reach a maximum value

$$\triangle V_{max} = \frac{\Omega B}{2c} r_p^2. \tag{10}$$

Let us make an estimate about the quark star's potential drop $\triangle V_q$ in the polar gap region:

$$\triangle V_q = \frac{\Omega B_q}{2c} r_{p,q}^2, \tag{11}$$

where $\Omega = 2\pi/P_q$ and $r_{p,q} = R_q(2\pi R_q/cP_q)^{1/2}$. For normal neutron stars, $\triangle V$ can be obtained just by changing the subscript q to NS. Thus

$$\frac{\triangle V_q}{\triangle V_{NS}} = \frac{B_q R_q^3 P_q^{-2}}{B_{NS} R_{NS}^3 P_{NS}^{-2}}. \tag{12}$$

Taking $R_q = 1$ km for a quark star with a mass of $0.001\,M_\odot$, $B_q = 10^8$ G, $B_{NS} = 10^{12}$ G, $R_{NS} = 10$ km, $P_q = 0.1$ ms and $P_{NS} = 1$ s, we find that $\triangle V_q/\triangle V_{NS} = 10$. This means that quark stars can have large enough potential drops in the polar cap regions.

In the inner gap model, the γ–B process plays a very important role, and two conditions should be satisfied at the same time for pair production: (1) to produce high-energy γ-ray photons, a high enough potential drop should be reached; (2) for pair production, the energy component of γ-ray photons perpendicular to the magnetic field must satisfy $E_{\gamma,\perp} \geq 2m_e c^2$ (Zhang & Qiao 1998).

Particles produced in the gap can be accelerated by the electric field in the gap, and the Lorentz factor of the particles can be written as

$$\gamma = \frac{e \triangle V}{m_e c^2}, \tag{13}$$

where γ is the Lorentz factor of the particles accelerated by the potential $\triangle V$, m_e is the mass of an electron or positron, and e is the charge of an electron.

In the $\gamma - B$ process, the conditions for pair production are that the mean free path of a γ-ray photon in the strong magnetic field should be equal to the gap height, $l \approx h$. The mean free path of a γ-ray photon is given by

$$l = \frac{4.4}{e^2/\hbar c} \frac{\hbar}{m_e c} \frac{B_c}{B_\perp} \exp\left(\frac{4}{3\chi}\right) \tag{14}$$

(Erber 1966), where $B_c = 4.414 \times 10^{13}$ G is the critical magnetic field, $\hbar$ is Planck's constant,

$$\chi = \frac{E_\gamma}{2m_e c^2} \sin\theta \frac{B}{B_c} = \frac{E_\gamma}{2m_e c^2} \frac{B_\perp}{B_c}, \tag{15}$$

and $B_\perp$ is the magnetic field perpendicular to the moving direction of γ photons, which can be expressed as (RS75)

$$B_\perp \approx \frac{h}{\rho} B \approx \frac{l}{\rho} B. \tag{16}$$

Here $l \approx h$ is the condition for sparks (pair production) to take place. ρ is the radius of curvature of the magnetic field lines. For a dipole magnetic configuration, it is

$$\rho \approx \frac{4}{3} (\lambda Rc/\Omega)^{1/2} \tag{17}$$

(Zhang et al. 1997a), where λ is a parameter used to describe the field lines, $\lambda = 1$ corresponding to the last opening field line. Gamma-ray energy from the curvature radiation process can be written as

$$E_{\gamma,CR} = \hbar \frac{3\gamma^3 c}{2\rho}. \tag{18}$$

We estimated the gap heights based on Zhang, Qiao & Han (1997b); that is,

$$h_{CR} \simeq 10^6 P^{3/7} B_8^{-4/7} \rho_6^{2/7} \text{ cm}. \tag{19}$$

When the relevant parameters used are $B = 10^8$ G, $P = P_q$, and assuming a dipole magnetic configuration, for any mass of quark stars, one can estimate the gap height from the curvature radiation (CR): $h_{CR} \approx 10^4\,\text{cm} = 100\,\text{m}$. This means that, even without the multipolar magnetic field assumption, the quark star can still fulfil the conditions for CR pair production.

There are three gap modes for pair production, namely the resonant ICS mode, the thermal-peak ICS mode and the CR mode (Zhang et al. 1997a). Each mode has its corresponding gap parameters, including the gap potential drop ΔV and the mean free path l of the $\gamma - B$ process. We estimated gap heights and other parameters based on the work of Zhang et al. (1997b), as shown in Table 2.

It can be seen from Table 2 that when the high-energy gamma-ray photons come from resonant photon production, the height of the gap is greater. For the thermal-peak ICS mode, it is one order of magnitude lower than for the CR mode, and two orders of magnitude lower than for the resonant ICS mode. This means that, in most cases, the thermal-peak ICS-induced pair production dominates in the gap.

The newborn submillisecond quark stars have high enough spin-down luminosities and gap potential drops (see Table 2) that they can emit radio or γ-ray photons with high enough luminosities to be detected by new facilities such as FAST and Fermi (formerly GLAST).

3.3 Lifetimes of submillisecond pulsars in the short-spin-period phase

Submillisecond pulsars may be very rare, or the time that such a pulsar stays in the short-period phase (<1 ms) may not be very long owing to magnetodipole (EM) radiation and gravitational wave (GW) radiation (Andersson 2003). The lowest-order GW radiation is bar-mode, which is caused by non-axisymmetric quadrupole moment. Here we consider GW radiation in the bar mode, which exerts a larger braking torque with braking index $n \approx 5$ than does magnetodipole radiation ($n = 3$). The rotation frequency drops quickly owing to GW radiation and EM radiation:

$$-I\Omega\dot{\Omega} = \frac{32GI^2\varepsilon_e^2\Omega^6}{5c^5} + \frac{B_0^2R^6\Omega^4}{6c^3}, \tag{20}$$

where c is the speed of light, $\varepsilon_e = \Delta a/\bar{a}$ is the gravitational ellipticity (equatorial ellipticity), Δa is the difference in equatorial radii and $\bar{a}$ is the mean equatorial radius.

To simplify equation (20), we introduce the notation $A = 32GI\varepsilon_e^2/(5c^5)$ and $D = B_0^2R^6/(6Ic^3)$, and integrate the equation in the angular velocity domain [$\Omega_i = 2\pi/P_i$, $\Omega_0 = 2\pi/0.001$]. Then

$$\tau = \frac{1}{2D}\left(\frac{1}{\Omega_0^2} - \frac{1}{\Omega_i^2}\right) - \frac{A}{2D^2} \ln \frac{\frac{1}{\Omega_0^2} + \frac{A}{D}}{\frac{1}{\Omega_i^2} + \frac{A}{D}}. \tag{21}$$

An accurate ellipticity of quark stars is unfortunately unavailable. Nevertheless, let us estimate ε_e to calculate the times in the submillisecond-period phase for GW and EM radiations. Cutler & Thorne (2002) suggested $\varepsilon_e = (I - I_0)/I_0 \leq 10^{-6}$. Regimbau & de Freitas Pacheco (2003) found from their simulations that $\varepsilon_e = 10^{-6}$ is the critical value to have at least one detection with first-generation interferometers (LIGO or VIRGO). It was shown that the direct upper limit was $\varepsilon_e \simeq 1.8 \times 10^{-4}$ on GW emission from the Crab pulsar using data from the first 9 months of the fifth science run of LIGO (Abbott et al. 2008). In addition, Owen (2005) showed that the maximum ellipticity of solid quark stars was $\varepsilon_{e,max} = 6 \times 10^{-4}$. From the on-line catalogue hosted by ATNF,[2] the seventh fastest rotating millisecond pulsar is PSR J0034-0534, which has a very low period derivative $\dot{P} \sim 4.96 \times 10^{-21}\,\text{s s}^{-1}$. We thus use such a low $\dot{P}$ and equation (20) to constrain the lower limit of the ellipticity of submillisecond pulsars, to $\varepsilon_{e,min} \sim 10^{-9}$ if the stellar mass is of the order of one solar mass. In order to facilitate a comparison of the lifetimes of quark stars and neutron stars (τ) in the submillisecond-period phase, we use mean equatorial ellipticities $\varepsilon_e = 10^{-6}$ and $\varepsilon_e = 10^{-9}$ to calculate τ for both quark stars and neutron stars through equation (21).

[2]http://www. atnf.csiro.au/research/pulsar/catalogue/ (Manchester et al. 2005)

Table 2. Gap parameters estimated for submillisecond quark stars. $\dot{E}_{\rm rot}$ is the spin-down luminosity; $h_{\rm CR}$ is the curvature radiation (CR) gap height; $\Delta V_{\rm CR}$ is the potential drop of the CR gap; $h_{\rm res}$ is the height of the resonant ICS gap; $\Delta V_{\rm res}$ is the potential drop of the resonant ICS gap; $h_{\rm th}$ is the thermal ICS gap height; and $\Delta V_{\rm th}$ is the potential drop of the thermal ICS gap.

M ($\mathrm{M}_{\odot}$)	$\dot{E}_{\rm rot}$(erg s^{-1})	$h_{\rm CR}$(cm)	$\Delta V_{\rm CR}$(V)	$h_{\rm res}$(cm)	$\Delta V_{\rm res}$(V)	$h_{\rm th}$(cm)	$\Delta V_{\rm th}$(V)
0.001	4.99×10^{36}	1.16×10^{4}	2.84×10^{10}	3.13×10^{5}	2.05×10^{13}	1.18×10^{3}	2.93×10^{8}
0.01	3.75×10^{38}	1.34×10^{4}	3.76×10^{10}	3.64×10^{5}	2.78×10^{13}	1.31×10^{3}	3.62×10^{8}
0.1	1.02×10^{40}	1.72×10^{4}	6.19×10^{10}	4.61×10^{5}	4.46×10^{13}	1.62×10^{3}	5.47×10^{8}
1	5.00×10^{40}	2.65×10^{4}	1.47×10^{11}	6.74×10^{5}	9.52×10^{13}	2.36×10^{3}	1.16×10^{9}

In the case of $\varepsilon_{\rm e} = 10^{-6}$, if we make the hypothesis that the rotational energy is lost because of EM radiation, then it can easily be derived that $\tau_{\rm EM} = 1/(2D)(1/\Omega_0^2 - 1/\Omega_i^2) \sim 5.9 \times 10^9$ yr for a typical compact star with $B_0 \sim 10^8$ G and $M = \mathrm{M}_{\odot}$. If, on the other hand, we suppose that the rotational energy is lost because of GW radiation, then $\tau_{\rm GW} = 1/(4A)(1/\Omega_0^4 - 1/\Omega_i^4) \sim 10^2$ yr for a typical compact star. The energy-loss rates of GW and EM radiation in the submillisecond-period phase, for a typical compact star that has a low magnetic field (10^8–10^9 G) either from AIC (Xu 2005) or spin up, are $\dot{E}_{\rm GW} = 32GI^2\varepsilon_{\rm e}^2\Omega^6/(5c^5) = 7.0\times10^{41}P_{\rm ms}^{-6}$ erg s^{-1} and $\dot{E}_{\rm EM} = B_0^2R^6\Omega^4/(6c^3) = 9.6\times10^{34}P_{\rm ms}^{-4}B_8^2R_6^6$ erg s^{-1}, respectively. Even if a quark star with 1 $\mathrm{M}_{\odot}$ formed from the AIC of a WD has a high magnetic field such as 10^{12} G, the lifetime τ in the submillisecond phase is 37 yr, in comparison with $\tau = 336$ yr for $B_0 = 10^8$ G. Then the EM energy loss is similar to the GW energy loss and becomes very important for $B_0 = 10^{12}$ G. For B_0 in the range from 10^8 G to 10^{11} G, it is always the case that $\dot{E}_{\rm GW} \gg \dot{E}_{\rm EM}$ for compact stars with short spin periods (<1 ms). Therefore, in the case of larger ellipticity (e.g. $\varepsilon_{\rm e} = 10^{-6}$), it is clear that GW radiation dominates the energy loss in the short-period phase for both recycled and AIC compact stars with low magnetic fields. The corresponding lifetime is shorter for a compact star with higher mass ($\sim\mathrm{M}_{\odot}$), but longer for a star with lower mass ($\sim$0.001 $\mathrm{M}_{\odot}$). However, if the ellipticity is lower, for example $\varepsilon_{\rm e} = 10^{-9}$, EM radiation dominates the rotational energy loss. The corresponding lifetime of a quark star (even with a high mass $\sim\mathrm{M}_{\odot}$) is long enough for us to detect it. Fig. 2 shows the relationship between lifetime (in the phase of <1 ms) and gravitation ellipticity $\varepsilon_{\rm e}$ for quark stars.

For the super-Keplerian case, the times in the phase of <0.5 ms for quark stars with various masses are calculated and listed in Table 1 (see τ_2). For a high-mass quark star with larger ellipticity, the time is too short for real detection, but the time is $>10^4$ yr for a low-mass quark star. Therefore, low-mass quark stars with $\varepsilon_{\rm e} \sim 10^{-6}$ could have much longer lifetimes in the phase of <0.5 ms. For a high-mass quark star with lower ellipticity, its lifetime is also very long in the phase of <0.5 ms. If a pulsar with spin period <0.5 ms is ever found, quark stars will be physically identified.

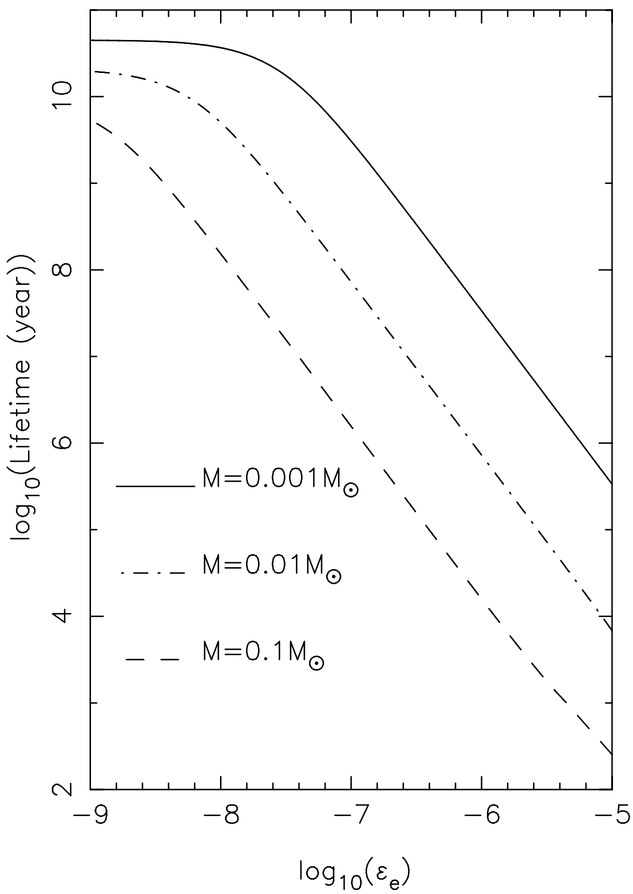

Figure 2. The relatioship between lifetime (in the phase with period <1 ms) and gravitational ellipticity $\varepsilon_{\rm e}$ for quark stars with masses of 0.001 $\mathrm{M}_{\odot}$ (solid line), 0.01 $\mathrm{M}_{\odot}$ (dot–dashed line), 0.1 $\mathrm{M}_{\odot}$ (dashed line), magnetic field $B = 10^8$ G and the bag constant $\beta = 60$ MeV fm^{-3}. The lifetime in the submillisecond-period phase is shorter if the mass of the quark star is higher.

3.4 Spin-down rate $\dot{P}$ for newborn quark and neutron stars

We also use equation (20) to calculate the period derivative ($\dot{P}$) for the nascent submillisecond quark stars and neutron stars.

Fig. 3 is a $\dot{P}$–P diagram that shows the spin-down evolution for quark stars with various masses. It is found that there are different properties for different ellipticities. For high ellipticities such as $\varepsilon_{\rm e} = 10^{-5}$, $\dot{P}$ can change by about 10 orders of magnitude for different periods (see the steep slopes of the dot–dashed lines and dashed lines). The rotational energy losses in this case are dominated by the GW radiation. For a low ellipticity such as $\varepsilon_{\rm e} = 10^{-9}$, in most cases the rotational energy losses are dominated by magnetic dipole (EM) radiation, and $\dot{P}$ changes relatively slowly with period (solid lines).

As a comparison, we also calculate the period derivative ($\dot{P}$) of a neutron star (with an initial period 0.5 ms, mass of 1.4 $\mathrm{M}_{\odot}$ and radius of 10^6 km). The results are shown in Fig. 4. It can be seen that $\dot{P}$ changes with period by as much as 10 orders of magnitude. It is found that the neutron star spins down much more quickly than low-mass quark stars, because of the neutron star's high mass ($\sim\mathrm{M}_{\odot}$), leading to a higher efficiency of GW radiation.

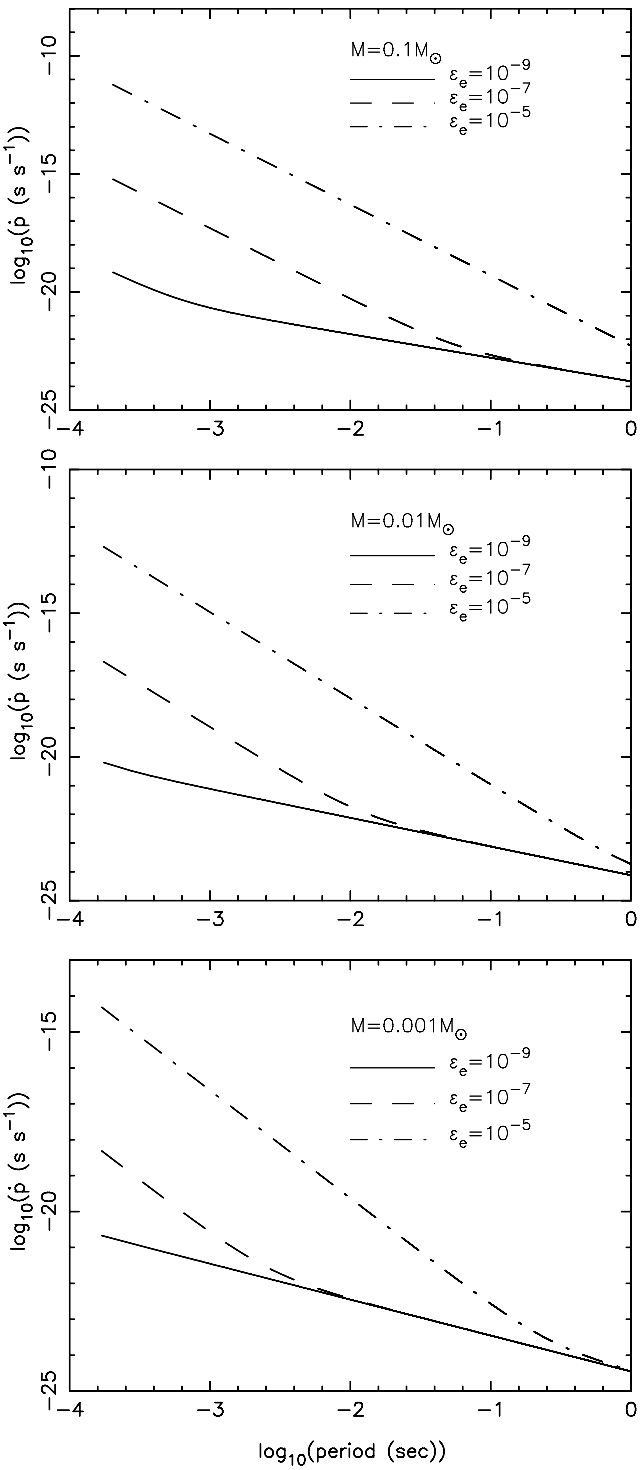

Figure 3. Spin-down evolution of quark stars owing to GW and EM radiation (period derivative versus spin period), for masses of 0.1 M$_\odot$, 0.01 M$_\odot$ and 0.001 M$_\odot$. We choose the ellipticity to be 10^{-5} (dot–dashed lines), 10^{-7} (dashed lines) and 10^{-9} (solid lines) in the calculation. It is evident that GW radiation dominates for quark stars with higher ε_e, whereas EM radiation dominates for lower ε_e.

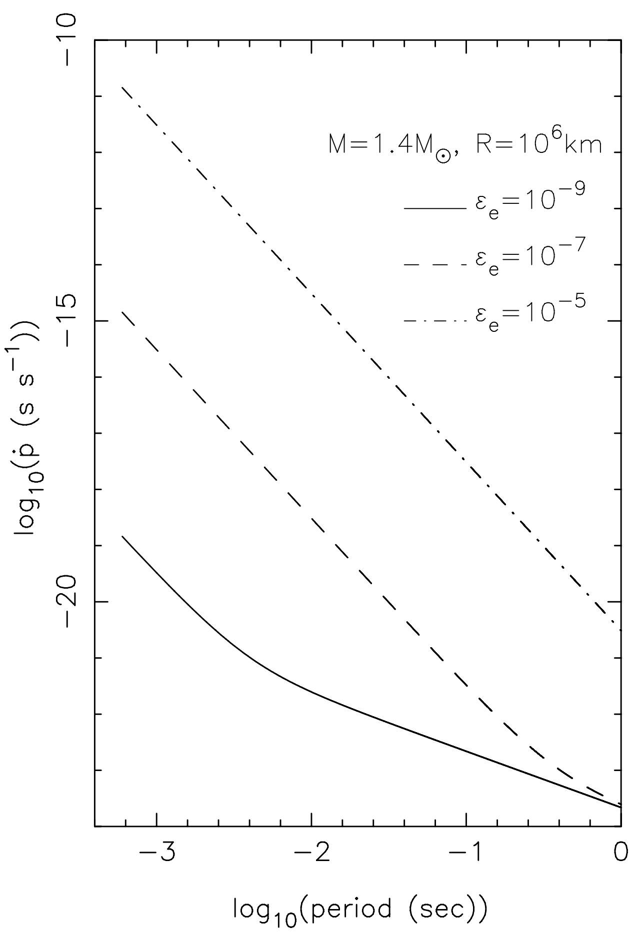

Figure 4. Period derivative versus spin period for a neutron star with an initial period of 0.5 ms, mass of 1.4 M$_\odot$ and radius of 10^6 km. The neutron star spins down quickly because of high mass (implying moment of inertia) for GW radiation.

4 SUBMILLISECOND PULSARS FORMED THROUGH ACCRETION IN BINARY SYSTEMS

There is also the important mechanism of 'spin up in binaries' for submillisecond pulsar formation, which is widely discussed in the literature. We regard this as the 'sub-Keplerian case' and make a comparison with our proposed AIC model 'super-Keplerian case'. In this section, we will find the minimal periods for both neutron stars and bare quark stars spun up by accretion in binary systems. We assume that the initial rotational periods of newborn pulsars have an 'equilibrium period' with two characteristic parameters: the magnetospheric radius and the corotation radius. The magnetospheric radius ($r_{\rm m}$) is the radius at which the ram pressure of particles is equal to the local magnetic pressure; that is,

$$r_{\rm m} = \phi R_A = \phi\left(\frac{4\mu_{\rm m}^2 M^{3/2}}{\dot{M}\sqrt{2G}}\right)^{2/7} = \phi\left(\frac{B_0^2 R^6}{\dot{M}\sqrt{2GM}}\right)^{2/7}$$
$$= \begin{cases} 3.24\times 10^8 \phi B_{12}^{4/7} M_1^{-1/7} R_6^{12/7} \dot{M}_{17}^{-2/7}\ \text{cm}, \\ 1.857\times 10^6 \phi B_8^{4/7} M_1^{3/7} \beta_{14}^{-4/7} \dot{M}_{17}^{-2/7}\ \text{cm}, \end{cases} \quad (22)$$

where $\mu_{\rm m}$ is the magnetic moment per unit mass of the compact star, B_8 is the surface magnetic strength in units of 10^8 G, $\dot{M}_{17}$ is the accretion rate in units of 10^{17} g s^{-1} and ϕ is the ratio between the magnetospheric radius and the Alfvén radius (Wang 1997; Burderi & King 1998). Wang (1997) studied the torque exerted on an oblique rotator and pointed out that ϕ decreased from 1.35 to 0.65 as the inclination angle increased from 0° to 90°. Here we take $\phi \sim 1$ (the influence of ϕ is discussed in Section 6).

Table 3. The minimal equilibrium period for quark stars and lifetimes due to GW and EM radiation in the submillisecond-period phase for quark stars with various masses ($10^{-3}\,\mathrm{M}_\odot, 0.1\,\mathrm{M}_\odot, 1.4\,\mathrm{M}_\odot$) in the sub-Keplerian case. τ_1, τ_2, τ_3 are calculated using $\varepsilon_e = 10^{-6}$, and $\tilde{\tau_1}, \tilde{\tau_2}, \tilde{\tau_3}$ are calculated using $\varepsilon_e = 10^{-9}$. The bag constant β is in units of Mev fm^{-3} and the accretion ratio α is in units of the Eddington accretion rate $\dot{M}_{\rm Edd}$.

β	α	$B_0(10^8$ G)	$P_{\rm eqmin}$(ms)	τ_1(yr)	$\tilde{\tau_1}$(yr)	τ_2(yr)	$\tilde{\tau_2}$(yr)	τ_3(yr)	$\tilde{\tau_3}$(yr)
60	0.71	1.1	0.613	2.9×10^7	2.8×10^{10}	1.3×10^4	4.1×10^9	1.7×10^2	1.5×10^8
110	0.85	1.4	0.453	5.1×10^7	3.6×10^{10}	2.3×10^4	5.6×10^9	2.3×10^2	2.6×10^8

Table 4. The minimal equilibrium period and lifetimes for GW and EM radiation in the submillisecond-period phase for various equations of state of normal neutron stars in the sub-Keplerian case. The mass and radius data for neutron stars were obtained from fig. 2 of Lattimer & Prakash (2004). τ and $\tilde{\tau}$ are times spent with a submillisecond period for neutron stars with $\varepsilon_e = 10^{-6}$ and $\varepsilon_e = 10^{-9}$, respectively.

EOS	$P_{\rm eqmin}$(ms)	Mass ($\mathrm{M}_\odot$)	Radius (km)	$B_0(10^8$ G)	$\dot{M}(10^{17}\,\mathrm{g\,s^{-1}})$	τ(yr)	$\tilde{\tau}$(yr)
AP4	0.55	2.21	10	2	6.36	103	1.02×10^8
GS1	0.52	1.38	8.27	2	5	255	2.37×10^8
PAL6	0.60	1.48	9.24	2	6.38	177	1.65×10^8
MS0	0.76	2.76	13.31	1	2.91	35	3.48×10^7
GM3	0.75	1.56	10.93	2	9.47	94	8.54×10^7
MS1	0.76	1.81	11.67	1	2.58	90	6.83×10^7

When $r_{\rm m}$ is very close to the radius of the compact star, we can rewrite the accretion rate $\dot{M}$ in units of the Eddington accretion rate ($\dot{M}_{\rm Edd}$), with a ratio, α, so that

$$\dot{M} = \alpha \dot{M}_{\rm Edd} = \alpha \frac{4\pi c m_{\rm p} R}{\sigma_T} = 1.0 \times 10^{18} \alpha M_1^{1/3} \beta^{-1/3}\ \mathrm{g\,s^{-1}}. \quad (23)$$

With these equations obtained above, we can obtain $r_{\rm m}$ for quark stars:

$$r_{\rm m} = 9.6 \alpha^{-2/7} B_8^{4/7} M_1^{1/3} \beta_{14}^{-10/21}\ \mathrm{km}. \quad (24)$$

The corotation radius is $r_{\rm c} = 1.5 \times 10^8 M_1^{1/3} P^{2/3}$ cm. The spin periods of compact stars cannot exceed the Kepler limit through accretion. When the compact star is spun up to the Kepler limit by the accreted matter falling onto the surface of the compact star, one can, for neutron stars, use the simple empirical relation for the maximum spin frequency as the equatorial radius expands:

$$\Omega_{\rm max} = 7700 M_1^{1/2} R_6^{-3/2}\ \mathrm{s^{-1}} \quad (25)$$

(Haensel & Zdunik 1989; Lattimer & Prakash 2004), which leads to

$$P_{\rm eq} \geqslant 0.816 M_1^{-1/2} R_6^{3/2}\ \mathrm{ms}, \quad (26)$$

where M and R refer to the mass and radius of the neutron star in non-rotating configurations.

For quark stars, Gourgoulhon et al. (1999) used a highly precise numerical code for the 2D calculations, and found that $\Omega_{\rm max}$ could be expressed as $\Omega_{\rm max} = 9920\sqrt{\beta_{60}}$ rad $\mathrm{s^{-1}}$, where $\beta_{60} = \beta/(60\,\mathrm{MeV\,fm^{-3}})$, which implies that $P_{\rm eq} \geqslant 0.633 \beta_{14}^{-1/2}$ ms. These are the so-called 'sub-Keplerian' conditions.

The accretion torque, N, exerted on the compact star is made up of two contributions: one is the positive material torque, which is carried by the material falling onto the star's surface; the other is the magnetic torque, which can be positive or negative, depending on the fastness parameter $\omega_s = \Omega_\star/\Omega_{\rm K} = (r_{\rm m}/r_{\rm c})^{3/2}$. It is suggested that the torques may cancel one another if the fastness parameter is $\omega_s = (r_{\rm m}/r_{\rm c})^{3/2} \approx 0.884$ (Dai & Li 2006). This implies a magnetospheric radius of $r_{\rm m} = 0.92\, r_{\rm c} \approx r_{\rm c}$. An equilibrium period of $P_{\rm eq}$ can be obtained when setting $r_{\rm m} = r_{\rm c}$:

$$P_{\rm eq} = \begin{cases} 0.512 B_8^{6/7} \beta_{14}^{-5/7} \alpha^{-3/7}\ \mathrm{ms}, & (27a) \\ 3170 B_{12}^{6/7} M_1^{-5/7} R_6^{18/7} \dot{M}_{17}^{-3/7}\ \mathrm{ms}. & (27b) \end{cases}$$

For quark stars, the equilibrium period is independent of mass and radius, and is dependent only on the bag constant, the surface magnetic field and the accretion rate. Take B_0 in the range $[10^8\,\mathrm{G}, 10^{12}\,\mathrm{G}]$: equation (27a) can be used to calculate the minimal equilibrium period for different EOSs for quark stars. For $\beta = 60\,\mathrm{MeV\,fm^{-3}}$, when $\alpha = 0.71$, $B_8 = 1.1$, the minimal period is 0.613 ms. For $\beta = 110\,\mathrm{MeV\,fm^{-3}}$, when $\alpha = 0.85$, $B_8 = 1.4$, the minimal period is 0.453 ms. (See the results in Table 3.)

For neutron stars, the data for the mass and radius for different EOSs were taken from Lattimer & Prakash (2004, their fig. 2) and B_0 is in the range $[10^8\,\mathrm{G}, 10^{12}\,\mathrm{G}]$. The minimal equilibrium period is calculated using equation (27b). (See the results in Table 4.)

In the sub-Keplerian case, the times spent in the submillisecond phase for quark stars of different masses and for neutron stars with different EOSs are listed in Tables 3 and 4, respectively. For typical quark stars as well as for neutron stars with high ε_e, their lifetimes in the submillisecond-period phase are about 10^2 yr, which results in a detection possibility that is too low. However, for low ε_e, the lifetime of a submillisecond pulsar (even with a high mass) is long enough for it to be detected.

5 CONCLUSIONS AND DISCUSSIONS

We have shown that, if a submillisecond pulsar is ever found, it could be a quark star, based on plausible scenarios for its origin, the energy available for radiation and its lifetime. A new possible way to form submillisecond pulsars (quark stars) through the AIC of WDs has been discussed in this paper. In the super-Keplerian case, we derived the initial period $P_{\rm q}$ through angular momentum conservation, taking into account special and general relativistic effects, and calculated the lifetime and gap parameters of a newborn quark star. Quark stars with different masses could have minimal rotational periods of about 0.1 ms. In most cases, quark stars would be bare (Xu 2002), and therefore a vacuum gap would be formed in the polar cap region. Based on our rough estimations, without considering the effect of frame dragging (Harding & Muslimov 1998), we found that the basic parameters (including rotational energy loss) in the gap are suitable for pair (electrons and positrons) production and sparking. These stars can be detected as submillisecond radio pulsars.

We also used an approximate formula to calculate the moment of inertia of the nascent quark star, but there are currently no accurate solutions for the configuration of rapidly rotating compact stars. This should be investigated in the future. In the calculation of WD mass and radius, we considered only the non-rotating configuration, but this does not change the conclusions of this paper. If the central density ρ_c of the WD is lower than 10^{11} g cm^{-3} before collapse, the resulting WD has a larger radius and moment of inertia; consequently, the newborn quark star could have a shorter spin period ($<$1 ms).

Both special and general relativistic effects are weak for a low-mass (e.g. Jupiter-like) quark star with a small radius. The rotational energy is lost through GW and EM radiation. The GW radiation dominates the rotational energy loss in the submillisecond-period phase, if the magnetic field of the star is not too large. Such quark stars therefore have a long time (several million years if the mass $\sim 10^{-3}$ M$_\odot$) with submillisecond spin periods. We have considered the bar mode of GW radiation in this paper, although other GW modes (e.g. the r-mode) that have not been considered here may be important (Xu 2006). The subsequent relaxation time-scale of a newborn quark star to a rigidly rotating configuration could be negligible, as a quark star may solidify soon after birth.

An important constraint for the detection of submillisecond pulsars is the lifetime in the phase of $<$1 ms due to GW and EM radiation. A possible method is proposed to constrain the lower limit of the pulsars' equatorial ellipticity (i.e. $\varepsilon_{\rm e,min} \sim 10^{-9}$), by evaluating the millisecond-pulsar period derivative from equation (20). For larger ellipticity, for example $\varepsilon_{\rm e} = 10^{-6}$, it is clear that GW radiation dominates the energy loss in the short-period phase for both recycled and AIC compact stars. The corresponding lifetime is shorter for a compact star with a higher mass ($\sim$M$_\odot$), but longer for a star with a lower mass ($\sim$0.001 M$_\odot$). However, if the ellipticity is lower, for example $\varepsilon_{\rm e} = 10^{-9}$, EM radiation dominates the rotational energy loss. The corresponding lifetime of a quark star (even with a high mass $\sim$ M$_\odot$) is long enough, and there are no lifetime constraints for the detection of submillisecond pulsars. Solid evidence of the existence of quark stars will be obtained if a pulsar with a period of less than $\sim$0.5 ms is discovered.

In the sub-Keplerian case, neutron and 'bare' quark stars can be spun up to submillisecond periods (even $\sim$0.5 ms) through accretion in binary systems. When neutron stars are spun up to the Kepler limit, the minimal equilibrium periods depend only on the mass and radius of the non-rotating configurations. The minimal equilibrium periods of quark stars depend on the bag constant.

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APPENDIX A: DIFFERENTIALLY ROTATING WD MODEL

The WD could be rotating differentially. As stated by Mueller & Eriguchi (1985), the WD's angular velocity Ω is a function of the distance from the rotation axis $\widetilde{\omega}$. The angular momentum distribution (the so-called rotation law) is

$$\Omega(\widetilde{r}) = \Omega_{\rm c}\frac{(aR_{\rm e})^2}{(aR_{\rm e})^2+\widetilde{r}^2}, \tag{A1}$$

where $\Omega_{\rm c}$ is the central angular velocity, $R_{\rm e}$ is the equatorial radius, and a is a free parameter. When differential rotation is taken into account, we can numerically evaluate the angular momentum of the inner collapsed core of the WD as

$$\begin{aligned} J_{\rm core} &= \sum_{\rm i} J_{\rm i} = \sum_{\rm i}\int_0^{\pi}\sigma 2\pi r_i^4 \sin^3\theta\,\Omega(r_i\sin\theta){\rm d}\theta \\ &= \sum_{\rm i}\left[\frac{m_{\rm core}\Omega_{\rm c}a^2R_{\rm WD}^2}{r_i^2}\right. \\ &\quad\left.\times\left(r_i^2-0.5\sqrt{\frac{r_i^2}{a^2R_{\rm WD}^2+r_i^2}}a^2R_{\rm WD}^2\ln\frac{1+\sqrt{\frac{r_i^2}{a^2R_{\rm WD}^2+r_i^2}}}{1-\sqrt{\frac{r_i^2}{a^2R_{\rm WD}^2+r_i^2}}}\right)\right], \end{aligned} \tag{A2}$$

where J_i is the angular momentum of each spherical shell with integral radius r_i.

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