PHYSICAL REVIEW D **85,** 023008 (2012)

Absorption features caused by oscillations of electrons on the surface of a quark star

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(Received 6 October 2011; published 23 January 2012)

If quark stars exist, they may be enveloped in thin electron layers (electron seas), which uniformly surround the entire star. These layers will be affected by the magnetic fields of quark stars in such a way that the electron seas would transmit hydromagnetic cyclotron waves, as studied in this paper. Particular attention is devoted to vortex hydrodynamical oscillations of the electron sea. The frequency spectrum of these oscillations is derived in analytic form. If the thermal x-ray spectra of quark stars are modulated by vortex hydrodynamical vibrations, the thermal spectra of compact stars (foremost, central compact objects and x-ray dim isolated neutron stars) could be used to verify the existence of these vibrational modes observationally. The central compact object 1E 1207.4-5209 appears particularly interesting in this context, since its absorption features at 0.7 keV and 1.4 keV can be comfortably explained in the framework of the hydrocyclotron oscillation model.

DOI: 10.1103/PhysRevD.85.023008 PACS numbers: 26.60.−c, 97.60.Gb, 97.60.Jd

I. INTRODUCTION

The spectral features of thermal x-ray emission are essential for us to understand the real nature of pulsarlike compact stars. Calculations show that atomic spectral lines form in the atmospheres of neutron stars. From the detection and identification of atomic lines in thermal x-ray spectra, one can infer neutron star masses (M) and radii (R), since the redshift and broadening of the spectral lines depend on M/R and M/R^2, respectively. Atomic features are expected to be detectable with the spectrographs on board of *Chandra* and *XMM-Newton*. No atomic features have yet been discovered with certainty, however. This may have its origin in the very strong magnetic fields carried by neutron stars. An alternative explanation could be that the underlying compact star is not a neutron star but a bare strange (quark matter) star [1,2]. The surface of such an object does not consist of atomic nuclei/ions, as is the case for a neutron star, but of a sea of electrons which envelope the quark matter.

Strange stars are quark stars made of absolutely stable strange-quark matter [3–8]. They consist of essentially equal numbers of up, down, and strange quarks, as well as of electrons [9–11]. The electrons are needed to neutralize the electric charges of the quarks, rendering the interior of strange stars electrically neutral. Quark matter is bound by the strong interaction, while electrons are bound to quark matter by the electromagnetic interaction. Since the latter is long-range, some of the electrons in the surface region of a quark star reside outside of the quark matter boundary, leading to a quark matter core which is surrounded by a fairly thin (thousands of femtometers thick) sea of electrons [8,9,12]. Because of the enormous advances in x-ray astronomy, more and more so-called dead pulsars, whose thermal radiation dominates over a very weak or negligibly small magnetospheric activity, are discovered. The best absorption features (at ∼0.7 keV and ∼1.4 keV) were detected for the central compact object (CCO) 1E 1207.4-5209 in the center of supernova remnant PKS 1209-51/52 (see Table I). Initially, these features were thought to be associated with the atomic transitions of ionized helium in a stellar atmosphere where a strong magnetic field is present [16]. Soon thereafter, however, it was noted that these lines are of electron-cyclotron origin [17]. The spectrum of 1E 1207.4-5209 shows two more features that may be caused by resonant cyclotron absorption, one at ∼2.1 keV and another, of lower significance, at ∼2.8 keV [18]. These features vary in phase with the star's rotation. Although the detailed mechanism which causes the absorption features is still a matter of debate, timing observations predict a rather weak magnetic field for this CCO, in agreement with what is obtained under the assumption that the lowest-energy line at 0.7 keV is the electron-cyclotron fundamental, favoring the electron-cyclotron interpretation [19,20]. Besides 1E 1207.4-5209, broad absorption lines have also been discovered in other dead pulsars (listed in Table I), especially in so-called x-ray dim isolated neutron stars (XDINSs), between about 0.3 and 0.7 keV [14].

In this paper, we reinvestigate the physics of these absorption features. The key assumption that we make here is that these features originate from the electron seas on quark stars rather than from neutron stars, whose surface properties are radically different from those of strange stars [8,9,12]. Of key importance is the magnetic field carried by a quark star, which critically affects the global properties (hydrodynamic surface fluctuations) of the electron sea at the surface of the star. We study this problem in the framework of classical electrodynamics in terms of cyclotron resonances of electrons in weak magnetic fields,

TABLE I. Dead pulsars (CCOs and XDINSs) with observed spectral absorption lines [13–15], with P as the spin period, B the magnetic field ($B_{10} = B/10^{10}$ G) derived by magnetodipole braking, T the effective thermal temperature detected at infinity, and E_a the absorption energy. We do not list the B fields of XDINSs for which the propeller braking could be significant because of their long periods.

Source	P/s	B_{10}	kT/keV	E_a/keV
RX $J0822.0-4300$	0.112	<98	0.4	⋯
1E $1207.4-5209$	0.424	<33	0.22	0.7, 1.4
CXOU $J185238.6+004020$	0.105	3.1	0.3	⋯
RX $J0720.4-3125$	8.39		0.085	0.27
RX $J0806.4-4123$	11.37		0.096	0.46
RX $J0420.0-5022$	3.45		0.045	0.33
RX $J1308.6+2127$	10.31		0.086	0.3
RX $J1605.3+3249$	⋯		0.096	0.45
RX $J2143.0+0654$	9.43		0.104	0.7

since the magnetic fields of dead pulsars are much lower than the critical field, $B_q = 4.414 \times 10^{13}$ G, at which the quantization of the cyclotron orbits of electrons into Landau levels occurs.

II. HYDROCYCLOTRON WAVES

A. Governing equations

For what follows, we restrict ourselves to a discussion of the large-scale oscillations of an electron sea subjected to a stellar magnetic field. We will be applying the semiclassical approach of the classical electron theory of metals and making use of standard equations of fluid mechanics. The electrons are viewed as a viscous fluid of uniform density $\rho = nm_e$ (where n is the electron number density) whose oscillations are given in terms of the mean electron flow velocity $\delta\mathbf{v}$. This implies that the fluctuation current-carrying flow is described by the density of the convective current $\delta\mathbf{j} = \rho_e\delta\mathbf{v}$, where $\rho_e = en$ is the electron charge density. The equations of motions of a viscous electron fluid are then given by [21]

$$\rho\frac{\partial\delta\mathbf{v}}{\partial t} = \frac{1}{c}[\delta\mathbf{j}\times\mathbf{B}] + \eta\nabla^2\delta\mathbf{v}, \tag{1}$$

$$\mathbf{j} = \rho_e\delta\mathbf{v}, \qquad \rho = m_e n, \qquad \rho_e = en, \tag{2}$$

where e and n are the change and number densities of electrons, respectively. We emphasize that $\delta\mathbf{j}$ stands for the convective current density and not for Ampère's $\mathbf{j} = (c/4\pi)\nabla\times\delta\mathbf{B}$, as is the case for magnetohydrodynamics. This means that the hydrodynamic oscillations in question are of non-Alfvén type. In Eq. (1), η denotes the effective viscosity of the electron fluid, which originates from collisions of electrons with the magnetic field lines at the stellar surface. It is worth noting that the cyclotron waves can be regarded as an analogue of the inertial waves in a rotating incompressible fluid, as presented in Eq. (III.56) of Chandrasekhar's book [22].

The governing equation, Eq. (1), can be represented as

$$\frac{\partial\delta\mathbf{v}}{\partial t} + \omega_c[\mathbf{n}_B\times\delta\mathbf{v}] - \nu\nabla^2\delta\mathbf{v} = 0, \tag{3}$$

$$\omega_c = \frac{eB}{m_e c}, \qquad \mathbf{n}_B = \frac{\mathbf{B}}{B}, \qquad \nu = \frac{\eta}{\rho}. \tag{4}$$

where ω_c is the cyclotron frequency. In the Appendix, we show that the electron sea can transmit macroscopic perturbations in the form of rotational hydrocyclotron waves which are characterized by the following dispersion relation:

$$\omega = \pm\omega_c\cos\theta\left[\frac{1}{1-(\nu k^2/\omega)^2} + i\frac{(\nu k^2/\omega)}{1-(\nu k^2/\omega)^2}\right], \tag{5}$$

where ω and k denote the frequency and wave vector of the perturbations, respectively. The Larmor radius of an electron in a strong magnetic field, $r_L \simeq m_e c^2/(eB) \propto B^{-1}$, is very small for pulsarlike compact stars, and we neglect the viscosity term in the following analysis of the motion of collective electrons. In the collision-free regime, $\nu = 0$, the hydrocyclotron electron wave is described as a transverse, circularly polarized wave whose dispersion relation and propagation speed are given by

$$\omega = \pm\omega_c\cos\theta \quad \text{and} \quad V = \pm(\omega_c/k)\cos\theta, \tag{6}$$

respectively. Here, θ is the angle between the magnetic field $\mathbf{B}$ and the wave vector $\mathbf{k}$. If $\mathbf{k}\parallel\mathbf{B}$, one has $\omega = \pm\omega_c$. In metals, these kinds of oscillations are observed as electron-cyclotron resonances. There are two possible resonance states, one for $\omega = \omega_c$ and the other for $\omega = -\omega_c$. These resonances correspond to the two opposite orientations of circularly polarized electron-cyclotron waves.

B. Hydrocyclotron oscillations of electrons on bare strange quark stars

We restrict our analysis to the collision-free regime of vortex hydrocyclotron oscillations. Using spherical coordinates, Eq. (3) then takes the form

$$\frac{\partial\delta\mathbf{v}}{\partial t} + \omega_c[\mathbf{n}_B\times\delta\mathbf{v}] = 0. \tag{7}$$

Taking the curl of both sides of Eq. (7), we obtain

$$\frac{\partial\delta\boldsymbol{\omega}}{\partial t} = \omega_c(\mathbf{n}_B\cdot\nabla)\delta\mathbf{v}, \qquad \delta\boldsymbol{\omega} = \nabla\times\delta\mathbf{v}. \tag{8}$$

Let the magnetic field $\mathbf{B}$ be directed along the z-axis, so that in Cartesian coordinates, $\mathbf{n}_B = (0, 0, 1)$. We then have

$$n_r = \cos\theta, \qquad n_\theta = -\sin\theta, \qquad n_\phi = 0. \tag{9}$$

From a mathematical point of view, the problem can be considerably simplified if one expresses the velocity $\delta\mathbf{v}$, which obeys the condition $\nabla \cdot \delta \mathbf{v} = 0$, in terms of Stokes' stream function, $\chi(\theta, \phi)$. This leads to

$$\delta v_r = 0, \qquad \delta v_\theta = \frac{1}{r\sin\theta}\frac{\partial \chi(\theta, \phi)}{\partial \phi},$$
$$\delta v_\phi = -\frac{1}{r}\frac{\partial \chi(\theta, \phi)}{\partial \theta}. \tag{10}$$

The depth of the electron layer near the star is much smaller than the stellar radius so that $r \approx R$ to a very good approximation. Equation (8) then simplifies to

$$\frac{\partial \delta\omega_r}{\partial t} = -\omega_c \frac{n_\theta \delta v_\theta}{R}, \tag{11}$$

with the radial component of the vortex given in terms of χ,

$$\delta\omega_r = \frac{1}{R}\left[\frac{\partial \delta v_\phi}{\partial \theta} - \frac{1}{\sin\theta}\frac{\partial \delta v_\theta}{\partial \phi}\right] = -\frac{1}{R^2}\nabla_\perp^2 \chi(\theta, \phi), \tag{12}$$

$$\nabla_\perp^2 = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}. \tag{13}$$

Substituting Eqs. (12) and (13) into Eq. (11) leads to

$$\nabla_\perp^2\left(\frac{\partial \chi}{\partial t}\right) + \omega_c \frac{\partial \chi}{\partial \phi} = 0. \tag{14}$$

The fact that free electrons undergo cyclotron oscillations in the planes perpendicular to the magnetic field suggests that the stream function χ can be written in the following separable form:

$$\chi(\theta, \phi) = \psi(\theta)\cos(\phi \pm \omega t). \tag{15}$$

The "+" sign allows for cyclotron oscillations which are induced by the clockwise polarized wave, and the "−" sign allows for cyclotron oscillations induced by the counterclockwise polarized wave. Substituting Eq. (15) into Eq. (14) leads to

$$\nabla_\perp^2 \psi(\theta) \pm \frac{\omega_c}{\omega}\psi(\theta) = 0. \tag{16}$$

In the reference frame where the polar axis is fixed, Eq. (16) is identical to the Legendre equation for the surface spherical function,

$$\nabla_\perp^2 P_\ell(\theta) + \ell(\ell+1)P_\ell(\theta) = 0, \tag{17}$$

where $P_\ell(\cos\theta)$ denotes the Legendre polynomial of degree ℓ. Hence, setting $\psi(\theta) = P_\ell(\theta)$, we obtain

$$\omega_\pm(\ell) = \pm\frac{\omega_c}{\ell(\ell+1)}, \qquad \omega_c = \frac{eB}{m_e c}, \qquad \ell \geq 1. \tag{18}$$

From this relation, we can read off the frequency of a surface hydrocyclotron oscillation of a given order ℓ.

TABLE II. Comparison between single-particle (Landau level) and hydrowave results for the cyclotron frequencies ($B_{12} = B/10^{12}$ G, $\omega_1 = \omega(\ell = 1)$, ω_c denotes the cyclotron frequency).

	$\omega(\ell=1)$	$\omega(\ell=2)$	$\omega(\ell=3)$	$\hbar\omega_1$/keV
Landau level	ω_c	$2\omega_c$	$3\omega_c$	$11.6B_{12}$
Hydrowave	$\omega_c/2$	$\omega_c/6$	$\omega_c/12$	$5.8B_{12}$

C. Characteristic features of cyclotron frequencies

Let us consider the spectrum of the positive branch $\omega(\ell) = \omega_+(\ell)$ of Eq. (18),

$$\frac{\omega(\ell)}{\omega_c} = \frac{1}{\ell(\ell+1)}, \qquad \ell \geq 1. \tag{19}$$

From

$$\frac{\omega(\ell)}{\omega(\ell+1)} = \frac{\ell+2}{\ell}, \qquad \ell \geq 1, \tag{20}$$

it follows that this ratio becomes a constant for $\ell \gg 1$. Such a spectral feature is notably different from the one of electron-cyclotron resonances of transitions between different Landau levels.

From the energy eigenvalues, E_n, of an electron in a strong magnetic field, which are found by solving the Dirac equation (see Ref. [23]), one may approximate the value of E_n for a relatively weak magnetic field, $B \ll B_q$, by

$$E_n = mc^2 + n\hbar\omega_c, \qquad n \geq 0. \tag{21}$$

Therefore, in the framework of a single-particle approximation, the emission/absorption frequencies, which are given by $\hbar\omega(\ell) = E_{n+\ell} - E_n$, should occur at

$$\omega(\ell) = \ell\omega_c, \qquad \ell \geq 1. \tag{22}$$

Table II compares the results of Eq. (22) with the results of Eq. (19) obtained for the hydrocyclotron wave model. Most notably, it follows that for the hydrocyclotron wave model, one obtains $\omega(\ell = 2)/\omega(\ell = 3) = 2$, in contrast to the cyclotron resonance model for single electrons which predicts this ratio for $\omega(\ell = 2)/\omega(\ell = 1) = 2$.

III. 1E 1207.4-5209 AND OTHER COMPACT OBJECTS

As already mentioned in the Introduction, 1E 1207.4-5209 (or J1210-5226) in PKS 1209-51/52 is one of the central compact objects in supernova remnants [15], where broad absorption lines, near (0.7, 1.4) keV [16] and possibly near (2.1, 2.8) keV [18], were detected for the first time. The interpretation of the absorption feature at ~2.8 keV is currently a matter of debate, in contrast to the feature at ~2.1 keV, which is essentially unexplained. Intriguingly, an absorption feature with the same energy, 2.1 keV, has also been detected in the accretion-driven x-ray pulsar 4U 1538-52 [24].

For what follows, we assume that 1E 1207.4-5209 is a strange-quark star and that (some of) these absorption features are produced by the hydrocyclotron oscillations of the electron sea at the surface of such an object. Assuming a magnetic surface field of $B \simeq 7 \times 10^{11}$ G and thus $\omega(\ell = 3) = 0.7$ keV, we obtain the oscillation frequencies shown in Table III. A magnetic field of $\sim 7 \times 10^{11}$ G is compatible with the magnetic fields inferred for 1E 1207.4-5209 from timing solutions [20] (9.9×10^{10} G or 2.4×10^{11} G), since 1E 1207.4-5209 shows no magnetospheric activity and the $\dot{P}$ value would be overestimated if one applies the spin-down power of magnetic-dipole radiation [25,26]. We note that the absorption feature at $\omega(\ell = 1) = 4.2$ keV shown in Table III may not be detectable since the stellar temperature is only $\sim$0.2 keV (see Table I), which will suppress any thermal feature in that energy range.

Aside from 1E 1207.4-5209, one may ask what would be the magnetic fields of other dead pulsars (e.g., radio-quiet compact objects) if their spectral absorption features would also be of hydrocyclotron origin. Intriguingly, the hydrocyclotron wave model predicts magnetic fields that are twice as large as those derived from the electron-cyclotron model if the absorption feature is at $\omega(\ell = 1) = \omega_c/2$; these fields could be $\sim$10 times greater (see Table II) if the absorption feature is at $\omega(\ell = 2) = \omega_c/6$ or $\omega(\ell = 3) = \omega_c/12$. The absorption lines at $(0.3 \sim 0.7)$ keV may indicate that the fields of XDINSs are on the order of $\sim 10^{10}$ to 10^{11} G, if oscillation modes with $\ell \geq 4$ are not significant.

As noted in Ref. [17], unique absorption features on compact stars are only detectable with *Chandra* and *XMM-Newton* if the stellar magnetic fields are relatively weak (10^{10} G to 10^{11} G), since the stellar temperatures are only a few tenths keV. The fields of many pulsarlike objects are generally greater than this value, with the exception of old millisecond pulsars whose fields are on the order of 10^8 G. Central compact objects, on the other hand, seem to have sufficiently weak magnetic fields (see Table I) so that absorption features originating from their surfaces should be detectable by *Chandra* and *XMM-Newton*. Arguments favoring the interpretation of compact central objects as strange quark stars have been put forward in Ref. [27], where it was shown that the magnetic field observed for some CCOs could be generated by small amounts of differential rotation between the quark matter core and the electron sea.

TABLE III. The frequencies, $\omega(\ell)$, at which hydrocyclotron oscillations occur for 1E 1207.4-5209 with effective temperature $T \simeq 0.2$ keV, assuming a magnetic field of $B \simeq 7 \times 10^{11}$ G.

ℓ	1	2	3	4	5	6
$\omega(\ell)$/keV	4.2	1.4	0.7	0.4	0.3	0.2

TABLE IV. Comparison between the frequencies, $\omega(\ell)$, and the absorption frequencies detected, $\omega_{\rm obs}$, for SGR 1806-20 (with an assumption of $B \simeq 1.86 \times 10^{13}$ G). The data of $\omega_{\rm obs}$ are from Ref. [29]. Both $\omega(\ell)$ and $\omega_{\rm obs}$ are in keV.

ℓ	1	2	3	4	5	6	7	8
$\omega(\ell)$	108	36	18	10.8	7.2	5.1	3.8	3.0
$\omega_{\rm obs}$	$\cdots$	$\cdots$	$17.5 \pm .5$	$11.2 \pm .4$	$7.5 \pm .3$	$5.0 \pm .2$	$\cdots$	$\cdots$

Besides dead pulsars, anomalous x-ray pulsars and soft gamma-ray repeaters (SGRs) are enigmatic objects which have become hot topics of modern astrophysics. Whether they are magnetars/quark stars is an open question [28]. In the case that anomalous x-ray pulsars/SGRs should be bare strange stars, the absorption lines detected from SGR 1806-20 could be understood in the framework of the hydrocyclotron oscillation model. Assuming a normal magnetic field of $B \simeq 1.86 \times 10^{13}$ G so that $\hbar\omega_c/12 = 18$ keV, one sees that the oscillation model predicts hydrocyclotron frequencies which coincide with the observation listed in Table IV.

IV. SUMMARY

In this paper, we studied the global motion of the electron seas on the surfaces of hypothetical strange-quark stars. It is found that such electron seas may undergo hydrocyclotron oscillations whose frequencies are given by $\omega(\ell) = \omega_c/[\ell(\ell + 1)]$, where $\ell \geq 1$ and ω_c is the cyclotron frequency. We propose that some of the absorption features detected in the thermal x-ray spectra of dead (e.g., radio silent) compact objects may have their origin in excitations of these hydrocyclotron oscillations of the electron sea, provided these stellar objects are interpreted as strange-quark stars. The central compact object 1E 1207.4-5209 appears particularly interesting. It shows an absorption feature at 0.7 keV which is not much stronger than the other absorption feature observed at 1.4 keV. This can be readily explained in the framework of the hydrocyclotron oscillation model, since two lines with ℓ and $\ell + 1$ could essentially have the same intensity. This is very different for the electron-cyclotron model, for which the oscillator strength of the first harmonic is much smaller than the oscillator strength of the fundamental.

ACKNOWLEDGMENTS

We would like to acknowledge valuable discussions at the PKU pulsar group. F. W. is supported by the National Science Foundation (USA) under Grant No. PHY-0854699. This work is supported by the National Natural Science Foundation of China (Grant Nos. 10935001 and 10973002), the National Basic Research Program of China

(Grant Nos. 2009CB824800 and 2012CB821800), and the John Templeton Foundation.

APPENDIX

Here, we derive the dispersion relation characterizing the propagation of hydrocyclotron electron waves in the slab-geometry approximation. The governing equation of viscous electron fluid under the action of Lorentz force is given by

$$\rho \frac{\partial \delta \mathbf{v}}{\partial t} = \frac{\rho_e}{c}[\delta \mathbf{v} \times \mathbf{B}] + \eta \nabla^2 \delta \mathbf{v},$$

which can be written as

$$\frac{\partial \delta \mathbf{v}}{\partial t} + \omega_c[\mathbf{n}_B \times \delta \mathbf{v}] - \nu \nabla^2 \delta \mathbf{v} = 0, \tag{A1}$$

where

$$\omega_c = \frac{eB}{m_e c}, \qquad \mathbf{n}_B = \frac{\mathbf{B}}{B}, \qquad \nu = \frac{\eta}{\rho},$$

$$\rho = m_e n, \qquad \rho_e = en,$$

ω_c is the cyclotron frequency, and η stands for the effective viscosity of an electron fluid originating from collisions between electrons. To make the problem analytically tractable, we treat the electron sea as an incompressible fluid and assume a uniform magnetic field. Equation (A1) can then be written as

$$\frac{\partial \delta \mathbf{v}}{\partial t} = -\omega_c[\mathbf{n}_B \times \delta \mathbf{v}] + \nu \nabla^2 \delta \mathbf{v}. \tag{A2}$$

Upon applying to Eq. (A2) the operator $\nabla \times$, we arrive at

$$\frac{\partial}{\partial t}[\nabla \times \delta \mathbf{v}] = \omega_c(\mathbf{n}_B \cdot \nabla)\delta \mathbf{v} - \nu \nabla \times \nabla \times \nabla \times \delta \mathbf{v}, \tag{A3}$$

where $\nabla \cdot \delta \mathbf{v} = 0$, and

$$\frac{\partial \delta \boldsymbol{\omega}}{\partial t} = \omega_c(\mathbf{n}_B \cdot \nabla)\delta \mathbf{v} - \nu \nabla \times \nabla \times \delta \boldsymbol{\omega}, \tag{A4}$$

where $\delta \boldsymbol{\omega} = \nabla \times \delta \mathbf{v}$. Considering a perturbation in the form of $\delta \mathbf{v} = \mathbf{v}' \exp[i(\mathbf{kr} - \omega t)]$, we have

$$[\mathbf{k} \times \delta \mathbf{v}] = i\frac{\omega_c}{\omega}(\mathbf{n}_B \cdot \mathbf{k})\delta \mathbf{v} + i\frac{\nu k^2}{\omega}[\mathbf{k} \times \delta \mathbf{v}], \tag{A5}$$

$$\left(1 - i\frac{\nu k^2}{\omega}\right)[\mathbf{k} \times \delta \mathbf{v}] = i\frac{\omega_c}{\omega}(\mathbf{n}_B \cdot \mathbf{k})\delta \mathbf{v}, \tag{A6}$$

where $(\mathbf{k} \cdot \delta \mathbf{v}) = 0$. It is convenient to rewrite the last equation as

$$[\mathbf{k} \times \delta \mathbf{v}] = i\frac{\omega_c}{\omega}(\mathbf{n}_B \cdot \mathbf{k})\delta \mathbf{v}\left[\frac{1 + i(\nu k^2/\omega)}{1 - (\nu k^2/\omega)^2}\right]. \tag{A7}$$

Multiplication of both sides of Eq. (A7) with $\mathbf{k}$ leads to

$$-\omega \delta \mathbf{v} k^2 = i\omega_c(\mathbf{n}_B \cdot \mathbf{k})\left[\frac{1 + i(\nu k^2/\omega)}{1 - (\nu k^2/\omega)^2}\right][\mathbf{k} \times \delta \mathbf{v}]. \tag{A8}$$

Inserting the left-hand side of Eq. (A7) into the right-hand side of Eq. (A8) gives

$$\omega^2 = \omega_c^2 \frac{(\mathbf{n}_B \cdot \mathbf{k})^2}{k^2}\left[\frac{1 + i(\nu k^2/\omega)}{1 - (\nu k^2/\omega)^2}\right]^2, \tag{A9}$$

or

$$\omega = \pm \omega_c \frac{(\mathbf{n}_B \cdot \mathbf{k})}{k}\left[\frac{1 + i(\nu k^2/\omega)}{1 - (\nu k^2/\omega)^2}\right],$$

which is Eq. (5).

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