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PULSAR BRAKING INDEX: A TEST OF EMISSION MODELS?

R. X. Xu[1,2] and G. J. Qiao[2,3]

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ABSTRACT

Pulsar braking torques due to magnetodipole radiation and the unipolar generator are considered, which results in a braking index n of less than 3 and could be employed to test the emission models. Improved equations for the pulsar braking index and magnetic field are presented, which are true if the rotation energy-loss rate equals the sum of the energy-loss rate of dipole radiation and of relativistic particles powered by a unipolar generator. The magnetic field calculated conventionally could be good enough, but only if it were modified by a factor of at most ∼0.6. Both inner and outer gaps may coexist in the magnetosphere of the Vela pulsar.

Subject headings: pulsars: general — radiation mechanisms: nonthermal

1. INTRODUCTION

The pulsar emission process is still poorly understood, even over 30 years after its discovery. Nevertheless, it is the consensus of researchers (e.g., Usov 2000) that primary pairs are produced and accelerated in regions (gaps) with a strong electric field along the magnetic line ($E_\parallel$) while more secondary pairs (with multiplicity $\sim10^2$–10^4) are created outside the gaps ($E_\parallel = 0$), and instability may be developed in the secondary $e^\pm$ relativistic plasma in order to give out coherent radio emission. Numerous models have been suggested concerning gap acceleration, and it is urgent to find an effective way to test those specific and detailed models against observations.

As a result of observational difficulties, only braking indices $n \equiv \Omega\ddot{\Omega}/\dot{\Omega}^2$ (Ω is the angular velocity of rotation) of five young radio pulsars have been obtained observationally (Lyne & Graham-Smith 1998 and references therein; Camilo et al. 2000). They are PSR B0531+21 ($n = 2.51 \pm 0.01$), PSR B1509−58 ($n = 2.837 \pm 0.001$), PSR B0540−69 ($n = 2.2 \pm 0.1$), PSR B0833−45 ($n = 1.4 \pm 0.2$), and PSR J1119−6127 ($n = 2.91 \pm 0.05$). These observed indices certainly include precious information on how pulsars produce radiation, but they are all remarkably smaller than the value of $n = 3$ expected for pure magnetodipole radiation, according to which the polar magnetic field strength at pulsar surface, B, is conventionally determined by (e.g., Manchester & Taylor 1977)

$$B = \frac{1}{\sin\alpha}\left(\frac{3Ic^3P\dot{P}}{8\pi^2R^6}\right)^{1/2}, \qquad (1)$$

where $P = 2\pi/\Omega$ is the rotation period, I the moment of inertia, c the speed of light, R the pulsar radius, and α the inclination angle. The term B is singular (i.e., $B \to \infty$) when $\alpha = 0°$. Thus, B-field derivation in this way is questionable and inconsistent since observation indicates that $n < 3$, which means that other processes do contribute to the braking torque. Indeed, some efforts appear to have found unusual torque mechanisms to which they contribute the observed braking index (see, e.g., Menou, Perna, & Hernquist 2001 and references therein).

An alternative effort, within the framework of "*standard*" neutron stars and their magnetospheric emission models, is proposed in this Letter. We find that n and B derivation should generally depend on pulsar emission models. Assuming that the orthogonal and aligned parts of magnetic moment are responsible for the dipole radiation and unipolar generator torques, respectively, we obtain consistent equations for calculating braking index and magnetic field in the inner vacuum gap, space charge–limited flow, and outer gap models. We find that all of these models result in a braking index of $n < 3$, and in return the models can be tested for a particular pulsar if its braking index and inclination angle are observed.

2. ASSUMPTION OF THE TOTAL ENERGY LOSS FOR ROTATION-POWERED PULSARS

Pulsar broadband emission depends essentially on a complete solution of the formidable well-defined magnetosphere problem in relativistic electrodynamics and plasma physics, which unfortunately is still unknown hitherto (e.g., Mestel 2000). Nevertheless, the problem has been understood to some extent in two particular cases, i.e., the orthogonal and aligned rotating cases.

Orthogonal rotator.—An orthogonal rotator with magnetic dipolar momentum $\mu_\perp$ emits monochromatic electromagnetic waves, the energy-loss rate of which is $\dot{E}_d = -2/(3c^3)\mu_\perp^2\Omega^4 \simeq -(6.2 \times 10^{27})B_{12}^2R_6^6\Omega^4$ ergs s^{-1}, where $B_{12} = B/(10^{12}$ G), $R_6 = R/(10^6$ cm), and $\mu_\perp = BR^3/2$. These low-frequency waves are generally unable to propagate and should be absorbed in neutron star surroundings, and a larger amount of energy and corresponding momentum could be pumped from neutron stars into their supernova remnants (Pacini 1967).

Aligned rotator.—The maximum potential drop in the open field line region by unipolar effect is (e.g., Ruderman & Sutherland 1975) $\triangle\Phi = \mu_\parallel\Omega^2/c^2 \simeq (5.56 \times 10^8)\,B_{12}R_6^3\Omega^2$ cgs. The $e^\pm$ pairs (or ions) are accelerated in charge-depletion gaps, picking up energy in the gaps and angular momentum from the magnetic torque when streaming out. The angular momentum loss requirement (Holloway 1977) can be satisfied if the charged particles can be "attached" to the magnetic field as far out or as near to the light cylinder. Two kinds of gaps are proposed to work in pulsar magnetospheres, termed inner and outer gaps. Various inner gaps that depend on the binding energy of charged particles in the pulsar surface are suggested, e.g., the vacuum gap model (Ruderman & Sutherland 1975), with enough binding, and the space charge–limited flow model, without any binding (Arons & Scharlemenn 1979; Harding & Muslimov 1998). The outer gap model was suggested to work near the null surface

[1] National Astronomical Observatories, Chinese Academy of Sciences, 20 Datun Road, Chaoyang, Beijing 100012, China.
[2] Department of Astronomy, Peking University, 5 Haidian Lu, Beijing 100871, China.
[3] CCAST, World Laboratory, P.O. Box 8370, Beijing 100080, China.

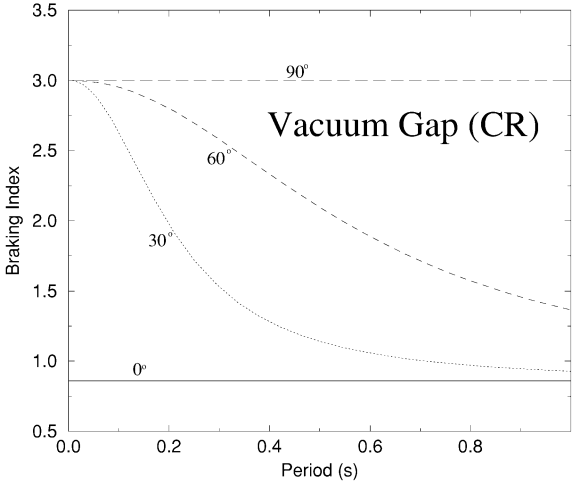

Fig. 1*a*

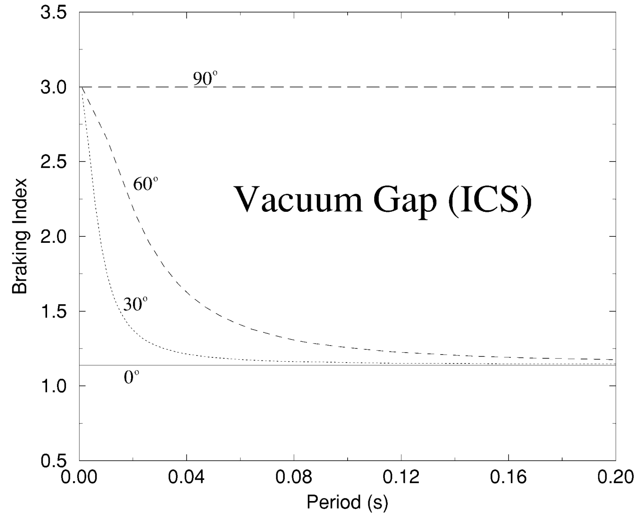

Fig. 1*b*

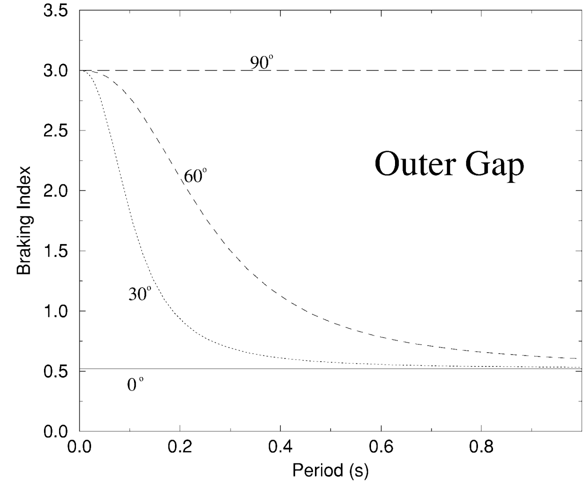

Fig. 1*c*

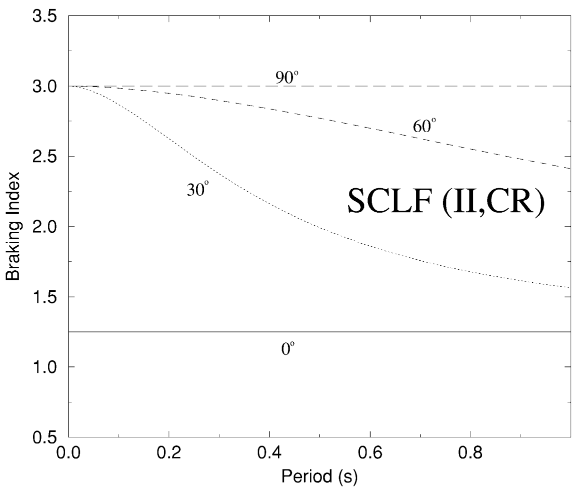

Fig. 1*d*

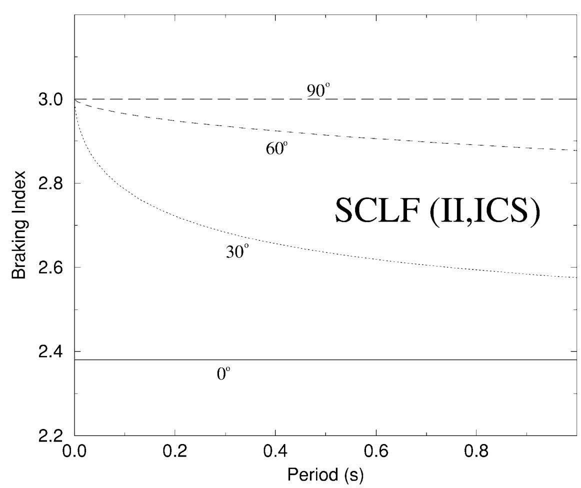

Fig. 1*e*

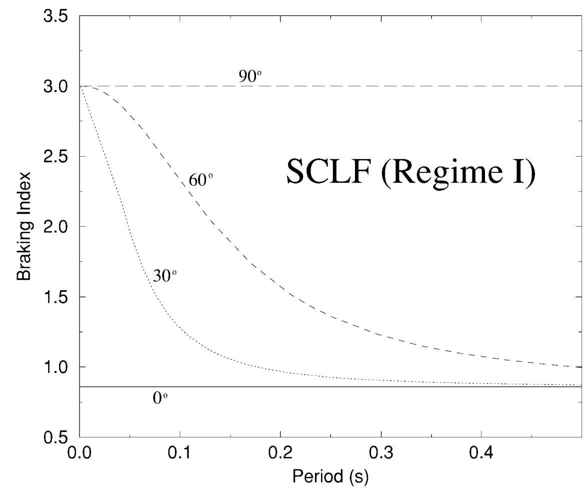

Fig. 1*f*

Fig. 1.—Set of calculated braking indices, as functions of rotation period, for six kinds of emission models. Pulsars are assumed to have polar magnetic field $B = 10^{12}$ G and radius $R = 10^{6}$ cm here. The inclination angles are chosen to be 0° (*solid lines*), 30° (*dotted lines*), 60° (*short-dashed lines*), and 90° (*long-dashed lines*). CR and ICS indicate CR-induced and resonant ICS-induced gaps, respectively. SCLF (regime I) : SCLF model without field saturation; SCLF (II, CR) and SCLF (II, ICS) : SCLF model with filed saturation (regime II).

TABLE 1
INCLINATION ANGLES (α) OF THE FIVE PULSARS DERIVED FROM MODELS

Name (PSR)	VG (CR) (deg)	VG (ICS) (deg)	OG (deg)	SCLF (II, CR) (deg)	SCLF (II, ICS) (deg)	SCLF (I) (deg)
B0531+21	2.6	2.9	5.0	2.1	1.6	33
B0540−69	2.5	6.3	5.2	1.8	…	36
B0833−45	2.6	6.9	6.7	1.0	…	31
B1509−58	11	8.4	26	7.6	2.5	81
J1119−6127	24	6.3	52	15	2.2	88

(e.g., Cheng, Ho, & Ruderman 1986; Zhang & Cheng 1997) because the charged particles on each side of the surface should flow in opposite directions in order to close a global current in a pulsar magnetosphere. Thus, it is obvious, as seen from above, that the energy loss is model dependent for aligned rotators, which will be considered when calculating pulsar braking indices and magnetic fields in § 3. Nevertheless, the energy-loss rate of an aligned rotator, due to unipolar effect, could be written in the form $\dot{E}_u = -2\pi r_p^2 c\varrho\Delta\phi$, if a gap has potential drop $\Delta\phi$ and the charge density in the gap is $\varrho = \zeta\varrho_{\rm gj} \approx \zeta(\Omega B/2\pi c) \simeq 5.3\zeta B_{12}\Omega$ cgs cm^{-3}, where the polar cap radius is $r_p = R\,(R\Omega/c)^{1/2} \simeq (5.77 \times 10^2)\,R_6^{3/2}\Omega^{1/2}$ cm; $\zeta \sim 1$ since ϱ and $\varrho_{\rm gj}$ are conventionally expected to be in a same order.

Assumption.—There are two schools of thought regarding the energy loss of an oblique magnetized rotator. One group opined that the magnetodipole radiation is the dominant mechanism of braking (e.g., Manchester & Taylor 1977; Dai & Lu 1998; Lyubarsky & Kirk 2001), where no braking appears when $\alpha = 0°$. Another group suggested that pulsars' spin-down dominates by a longitudinal current outflow due to the unipolar generator (e.g., Beskin, Gurevich, & Istomin 1984), where Ω is constant if $\alpha = 90°$. However, although there are two unseemly points when $\alpha = 0°$ for the first school and when $\alpha = 90°$ for the second school, an interesting and strange thing, explained in § 3, is that the derived physical parameters (e.g., *B*-field strength) are reasonable. We propose that both energy-loss mechanisms above, i.e., via dipole radiation and the unipolar generator, are expected to contribute the total braking torque of an oblique pulsar.

Phenomenologically, for a pulsar with a total magnetic momentum $\boldsymbol{\mu} = \boldsymbol{\mu}_\perp + \boldsymbol{\mu}_\parallel$ ($\mu_\perp = \mu\sin\alpha$, $\mu_\parallel = \mu\cos\alpha$), we could write the total energy loss in the form $\dot{E} = c_\perp\dot{E}_d + c_\parallel\dot{E}_u$, where $c_\perp$ and $c_\parallel$ are generally two functions of α indicating the contributions of those two energy-loss mechanisms, respectively. Certainly, $c_\perp(\alpha = \pi/2) = 1$ and $c_\parallel(\alpha = 0) = 1$. An essential and simple assumption employed in this Letter is that $c_\perp = c_\parallel = 1$ since $\dot{E} = \dot{E}_d + \dot{E}_u$ if $\mu_\perp$ and if $\mu_\parallel$ result *independently* in spin-downs of $\dot{E}_d$ and $\dot{E}_u$, respectively. Therefore, we have $\dot{E} = -2\mu^2/(3c^3)\,\Omega^4\eta$, with $\eta \equiv \sin^2\alpha + 3\cos^2\alpha\,[\Delta\phi/(\Delta\Phi)] \simeq \sin^2\alpha + (5.4 \times 10^{-9})R_6^{-3}B_{12}^{-1}\cos^2\alpha \times \Omega^{-2}\Delta\phi$.

3. BRAKING INDEX AND ITS IMPLICATION

The energy carried away by the dipole radiation ($\dot{E}_d$) and the relativistic particles ($\dot{E}_u$) originates from the rotation kinetic energy, the loss rate of which is $\dot{E} = I\Omega\dot{\Omega}$. Energy conservation conduces toward

$$\dot{\Omega} = -\frac{2\mu^2}{3c^3 I}\Omega^3\eta. \tag{2}$$

Based on equation (2), the braking index can be derived to be

$$n = 3 + \frac{\Omega\dot{\eta}}{\dot{\Omega}\eta} = 3 + \frac{\Omega}{\eta}\frac{d\eta}{d\Omega}, \tag{3}$$

which is not exactly equal to 3 as long as η is not a constant. If $\eta \propto \Omega^a$, then $n < 3$ for $a < 0$ ($n > 3$ for $a > 0$). For pulsars near the death line, $\Delta\phi \simeq \Delta\Phi$; i.e., the maximum potential drop available, $\Delta\Phi$, acts on the gap. In this case, $\eta = 1 + 2\cos^2\alpha < 3$ and $\dot{\eta} = -2\sin(2\alpha)\dot{\alpha}$. So $n < 3$ if α gets smaller as a pulsar evolves. For pulsars away from the death line, the potential drop, $\Delta\phi$, across an accelerator gap, which is model dependent, is much smaller than $\Delta\Phi$. We discuss baking index in the following models, assuming that $\boldsymbol{\mu}$ (μ and α) and I are not changed for simplicity since both observation (Bhattacharya et al. 1992) and theory (e.g., Xu & Busse 2001) imply that a pulsar's *B*-field does not decay significantly during the rotation-powered phase.

Vacuum gap (*VG*) *model.*—The basic picture of the vacuum gap formed above the polar cap with enough binding energy was delineated explicitly in Ruderman & Sutherland (1975), where relativistic primary electrons emit γ-rays via curvature radiation (CR) in the gap. The gap potential difference $\Delta\phi_{\rm CR}^{\rm VG} = (4.1 \times 10^9)\,\rho_6^{4/7}B_{12}^{-1/7}\Omega^{1/7}$ cgs, where the curvature radius[4] is $\rho = \rho_6 \times 10^6$ cm. For polar cap accelerators, $\rho \simeq (4/3)\,(Rc/\Omega)^{1/2} \approx (2.3 \times 10^8)\,R_6^{1/2}\Omega^{-1/2}$. We thus have $\Delta\phi_{\rm CR}^{\rm VG} = (9.2 \times 10^{10})\,R_6^{2/7}B_{12}^{-1/7}\Omega^{-1/7}$ cgs and $\eta_{\rm CR}^{\rm VG} \simeq \sin^2\alpha + (4.96 \times 10^2)\,R_6^{-19/7}B_{12}^{-8/7}\cos^2\alpha\Omega^{-15/7}$. For a vacuum gap where primary electrons emit γ-rays via resonant inverse Compton scattering (ICS) off the thermal photons (e.g., Zhang, Harding, & Muslimov 2000), the potential drop and the η-value are $\Delta\phi_{\rm ICS}^{\rm VG} = (1.9 \times 10^{13})\,R_6^{4/7}B_{12}^{-15/7}\Omega^{1/7}$ cgs and $\eta_{\rm ICS}^{\rm VG} \simeq \sin^2\alpha + (1.02 \times 10^5)\,R_6^{17/7}B_{12}^{-22/7}\cos^2\alpha\Omega^{-13/7}$.

Space charge–limited flow (*SCLF*) *model.*—The SCLF model works for pulsars with a boundary condition of $E_\parallel = 0$ at the pulsar surfaces. The previous SCLF (Arons & Scharlemann 1979) model has been improved to a new version (e.g., Harding & Muslimov 1998) that includes the frame-dragging effect. Although a simple and general analytical formula for all pulsars is not available in the Harding & Muslimov (1998) model, the potential drop could be well approximated in the extreme cases, regimes I and II, which are defined, respectively, as cases without or with field saturation.[5] In the regime II case (i.e., the gap height is larger than r_p), Zhang et al. (2000) obtained the potential drop according to which η-values can be calculated. For the CR-induced SCLF models, $\Delta\phi_{\rm II,\,CR}^{\rm SCLF} = (7.1 \times 10^9)\,R_6^{3/4}\Omega^{1/4}$ cgs and $\eta_{\rm II,\,CR}^{\rm SCLF} \simeq \sin^2\alpha + 38R_6^{-9/4}B_{12}^{-1}\cos^2\alpha\Omega^{-7/4}$; for the resonant ICS-induced SCLF models, $\Delta\phi_{\rm II,\,ICS}^{\rm SCLF} = (4.2 \times 10^8)R_6^{28/13}B_{12}^{-9/13} \times \Omega^{18/13}$ cgs and $\eta_{\rm II,\,ICS}^{\rm SCLF} \simeq \sin^2\alpha + 2.3R_6^{-11/13}B_{12}^{-22/13}\cos^2\alpha\Omega^{-8/13}$. In regime I, the stable acceleration scenario should be controlled by curvature radiation (Zhang & Harding 2000), $\Delta\phi_{\rm I}^{\rm SCLF} = (1.8 \times 10^{11})R_6^{4/7}B_{12}^{-1/7}\Omega^{-1/7}$ cgs and $\eta_{\rm I}^{\rm SCLF} \simeq \sin^2\alpha + 9.8 \times R_6^{-17/7}B_{12}^{-8/7}\cos^2\alpha\Omega^{-15/7}$.

[4] Ruderman & Sutherland (1975) supposed that there are multipole magnetic fields near pulsar surfaces, and thus they had $\rho_6 = 1$. But in this Letter we simply use dipole field lines for indication.

[5] The definitions of regimes I and II in Zhang & Harding (2000) were misprinted (B. Zhang 2001, private communication).

Outer gap (*OG*) *model*.—For a self-sustaining outer gap, which is limited by the $e^{\pm}$ pair produced by collisions between high-energy photons from the gap and soft X-rays resulting from the surface heating by the backflowing primary $e^{\pm}$ pairs, the potential drop is $\Delta\phi = f^2\Delta\Phi$, where the fractional size of such an outer gap is $f = 5.5B_{12}^{-4/7}P^{26/21}$ (Zhang & Cheng 1997). Here $f < 1$, which is satisfied for the five pulsars if the outer gap works. The η-value therefore can be calculated, $\Delta\phi^{\rm OG} = (1.59 \times 10^{12})\, R_6^3 B_{12}^{-1/7}\Omega^{-10/21}$ cgs and $\eta^{\rm OG} \simeq \sin^2\alpha + (8.6 \times 10^3)B_{12}^{-8/7}\cos^2\alpha\Omega^{-52/21}$.

From these η-values in different models, the braking index can be obtained by equation (3). For typical pulsars with $R_6 = 1$ and $B_{12} = 1$, we compute the braking index n in each model, which is shown in Figure 1. It is obvious that $n < 3$ as long as inclination angle $\alpha < 90°$ in all of the models. Pulsars with small rotation periods tend to have $n \approx 3$. Also, we can see from Figure 1 or equation (2) that there is a minimum braking index $n(\alpha = 0°)$ for each model. In the case of $B_{12} = R_6 = 1$, $n_{\rm CR}^{\rm VG}(\alpha = 0°) = 0.86$, $n_{\rm ICS}^{\rm VG}(\alpha = 0°) = 1.14$, $n^{\rm OG}(\alpha = 0°) = 0.52$, $n_{\rm II,\,CR}^{\rm SCLF}(\alpha = 0°) = 1.25$, $n_{\rm II,\,ICS}^{\rm SCLF}(\alpha = 0°) = 2.38$, and $n_{\rm I}^{\rm SCLF}(\alpha = 0°) = 0.86$.

We cannot solve out magnetic field B only by equation (2) because $\eta = \eta(\alpha, \Omega)$. If $\alpha = 90°$ (or $\eta = 1$), the solution of equation (2) results in equation (1). In principal, equations (2) and (3) should be combined to find consistent B and α in the case of known braking index. However, because $1 < \eta < 3$, the magnetic field derived from equation (1) is good enough but is modified by a factor of only $1/\sqrt{\eta} \in (0.58, 1)$.

Based on equations (2) and (3), the inclination angles of the five pulsars with observed braking indices are calculated in different models (see Table 1). No solution of α is available for the Vela pulsar (PSR B0833−45) and PSR B0540−69 for the regime II SCLF (ICS) model since their braking indices are smaller than $n_{\rm II,\,ICS}^{\rm SCLF}(\alpha = 0°)$. This is consistent with the fact that these pulsars are young and their gap heights are thus much smaller than r_p.

Furthermore, we can determine whether a model works on a particular pulsar by comparing the calculated α in Table 1 with the observed α. Usually, α can be derived by fitting the position angle curves of pulsars with high linear polarization in the rotating vector model (Lyne & Manchester 1988). For the five pulsars, only the inclination angle of the Vela pulsar is obtained (~90°); however, no α-value in Table 1 tallies with this observation. There are two possible explanations for the discrepancy: (1) The braking torques due to the dipole radiation and the unipolar generator should be treated and added in a manner other than ours (e.g., Harding, Contopoulos, & Kazanas 1999). However, our treatment of the torques is reasonable, so further improvement of the braking calculation might not substantially change the results presented. (2) No model listed in Table 1 can perfectly describe the actual accelerating situation of the Vela pulsar. The outer gap model explains well the high-energy emission of this pulsar but could still be a partial description of the global magnetosphere. One possible picture is that both the inner and outer gaps coexist in a pulsar's magnetosphere (Usov 2000), but the *interaction* between these two gaps and the pair plasma properties is still very uncertain. It is also possible that the pair production process in strong magnetic and electric fields should be improved. For example, if $B > 0.1B_c$ ($B_c = 4.4 \times 10^{13}$ G), γ photons nearly along curved field lines convert into positroniums, which could partially prevent the screening of $E_{\parallel}$ (thus increasing the gap height and possibly having $\zeta > 1$), and therefore the energy loss $\dot{E}_u$ increases significantly in polar cap models (Usov & Melrose 1996). Such an increase could result in a larger α in Table 1 (see eq. [3]) since all magnetic fields of the five pulsars are very strong (near or greater than $0.1B_c$). In conclusion, further studies that test emission models via braking index and that analyze the theoretical meanings of the test results would be interesting and necessary.

4. CONCLUSION AND DISCUSSION

We have proposed in this Letter that the observed braking index $n < 3$ can be understood if the braking torques due to dipole radiation and the unipolar generator are combined. The discrepancy between the observed inclination angle and that derived from the six models of the Vela pulsar in Table 1 may call for improved pulsar emission models. In addition, it is found that the magnetic field strength of a pulsar by conventional method could be a pretty good representation of the actual one.

Figure 1 shows the variations of braking index n as functions of pulsar periods. Since pulsars spin down in their life, the curves in Figure 1 represent the variations of n as functions of pulsar ages to some extent; n decreases as a pulsar evolves. However, the Johnston & Galloway (1999) method of deriving the braking index can be applied only if n is constant during pulsar life. Therefore, in principle, n cannot be obtained by only P and $\dot{P}$.

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