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To probe into pulsar's interior through gravitational waves

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Abstract

The gravitational radiation from compact pulsar-like stars depends on the state of dense matter at supranuclear densities, i.e., the nature of pulsar (e.g., either normal neutron stars or quark stars). The solid quark star model is focused for the nature of pulsar-like compact objects. Possible gravitational emission of quark stars (either fluid or solid) during the birth and later lifetime is discussed. Several observational features to distinguish various models for pulsar-like stars are proposed. It is suggested that the gravitational wave behaviors should be mass-dependent. Based on the data from the second LIGO science run, the upper limits of $R \cdot \theta^{1/5}$ and thus $M \cdot \theta^{3/5}$ (M: mass, R: radius, and θ: wobble angle) are provided for those targets of millisecond pulsars.
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1. Introduction

On the one hand, it is now a very critical time for detecting *directly* gravitational waves. Despite the negative detection results, some limits on the physical parameters of pulsar-like star's interiors can be obtained with theoretical calculations which relate the waves to star's nature in literatures, based on the experiment data of gravitational waves. Andersson and Kokkotas [4] calculated the eigenfrequencies of the *f*, *p*, and *w* modes, and found that the frequencies depend on star's mass and radius. Numerical results [6] showed that the information of neutron star structure could be carried by gravitational waves of the axial *w*-modes. The inverse problem for pulsating neutron stars, based on the studies of those modes, was discussed extensively by Kokkotas et al. [17]. The frequencies and damping behaviors were reexamined for different equations of state [7], and recently, Tsui and Leung [33] applied an inversion scheme to determine star's mass, radius, and density distribution via gravitational wave asteroseismology. From the theoretical work above, one sees then that a positive result of recording gravitational wave signal should improve significantly the knowledge of matter at supranuclear density.[1]

On the other hand, pulsar-like compact stars were discovered since 1968, but, unfortunately, we are still not sure about their nature. It is conventionally believed that these compact stars are normal neutron stars. However one can *not* rule out the possibility, that they are actually quark stars composed of quark matter with de-confined quarks and gluons, either from the first principles or according to their various observations [14,20,34]. Quarks are fundamental Fermions in the standard model of particle physics, and the existence of quark matter is a direct consequence of the asymptotically free nature of the strong interaction, which was proved in the regime of non-Abelian gauge theories and confirmed by high-energy collider experiments. In

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[1] It may be worth noting, however, that gravitational wave or graviton might not exist at all in the de Sitter Universe [23] if gravitational wave would still not be detected in the next decades.

this sense, to affirm or negate the existence of quark stars is of significantly fundamental meaning for understanding the nature's elementary strong interaction. Quark stars with strangeness are popularly discussed in literatures, which are called as strange (quark) stars, whereas non-strange quark stars would also be possible if non-strange quark matter could also be stable at zero pressure due to color confinement [25]. We do not differentiate between the terms of quark stars and strange stars here, supposing both kinds of the stars have a quark surface to confine quarks and gluons.

Therefore, one of the key points for today's astrophysicists is understanding the nature of pulsars: to find competitive evidence for quark stars or for normal neutron stars. Effective methods to do depend on (i) the minimum spin periods, (ii) the mass–radius relations, and (iii) the surface differences [37]. However, since they are relativistic, pulsar-like compact stars could be potentially gravitational wave radiators [44]. The features and strength of this emission would reflect their nature, and one may probe into pulsar's interior with gravitational waves. Recently, the Laser Interferometric Gravitational wave Observatory (LIGO) has monitored 28 radio pulsars [1], and obtained upper limits of gravitational radiative intensities for the pulsars. If pulsars are quark stars, how and what can one constraint the pulsar's nature based on the data? We are trying to analyze relevant issues in this paper.

There are actually two motivations to study gravitational waves using pulsars: (i) as a tool and (ii) as a source. These two result in different physics. The radio pulsar timing array depends on the "clock" nature, not on the internal structure of pulsars, while the direct detection of the waves from isolated and/or binary pulsars depends on the internal structure of pulsars. The former is not discussed in this paper. Certainly, a successful detection of gravitational waves from pulsar-like stars is very important for understanding (i) the physics of matter at supranuclear density and (ii) the physics of gravity.

2. Quark stars during their birth

A mass formula for strangelet analogous to the Bethe-Weizsacher semi-empirical mass function in nuclear physics was introduced by Desai et al. [11]. Neglecting the symmetry and Coulomb energy terms, which are not important in the case of strange quark stars, one has $E = E_v + E_s$, where the volume and surface energy contributions are $E_v = \epsilon_0 A'$ and $E_s \sim 0.1\epsilon_0 A'^{2/3}$, respectively, with $\epsilon_0 = 880$–890 MeV and A' the baryon number [11]. By introducing baryon number density $n_b = A'/(4\pi R^3/3)$ (R the radius of spherical quark matter) and the surface energy per unit area γ, we have $R = (3A'/(4\pi n_b))^{1/3}$ and thus $\gamma \simeq 0.02\epsilon_0 n_b^{2/3} \simeq 20$ MeV fm^{-2}. However, Bagchi et al. [5], obtained a surface energy (or surface tension), γ, which varies from about 10 to 140 MeV fm^{-2}, depending on stellar radius. It is worth noting that the surface energy, which will be neglected in this paper, due to an electrostatic field of $<\sim10^{17}$ V/cm on a star's surface is several orders smaller that 10 MeV fm^{-2}. We consider thus this poorly known parameter γ to range between $\sim10^1$ MeV fm^{-2} and $\sim10^2$ MeV fm^{-2}.

2.1. Quark nuggets ejected

A hot turbulent bare quark star may eject (or "evaporate") low-mass quark matter (quark nuggets). Phenomenologically, there could be 4 steps to create a quark nugget (Fig. 1). Two parameters, which are not know with certainty yet, describe the turbulent nature of protostrange stars: the convective scale l_t and velocity v_t. It is possible that $l_t \sim 10^{2-5}$ cm, $v_t \sim 10^5$ cm/s [40]. In step (a), the typical kinematic energy of turbulent part is $E_t \sim \rho l_t^3 v_t^2$, while the surface and the gravitational energies, which may prevent the fluid part to eject, are $E_s \sim \gamma l_t^2$ and $E_g \sim GmMr/R^2$ (quark star's mass M and its radius R, lump's mass m and radius r), respectively. The height of a bulge increases if $E_s + E_g < E_t$, as is shown in step (b). The contact area between the star and the bump becomes smaller and smaller, as shown in step (c), because of (i) a limited scale of kinematic flow with order, and (ii) a total momentum which tends to separate nugget from star. A quark nugget form finally, with a velocity of $V_{qn} <\sim v_t$, step (d). The density of a bare strange star with mass $\leqslant\sim M_\odot$ is nearly uniform [2], and the mass of such a star can be well approximated by

$$M \simeq \frac{4}{3}\pi R^3 (4B), \tag{1}$$

where the bag constant $B = 60$–110 MeV fm^{-3}, i.e., $(1.07$–$1.96)\times10^{14}$ g/cm^3. For simplicity, we apply this approximation throughout this paper.
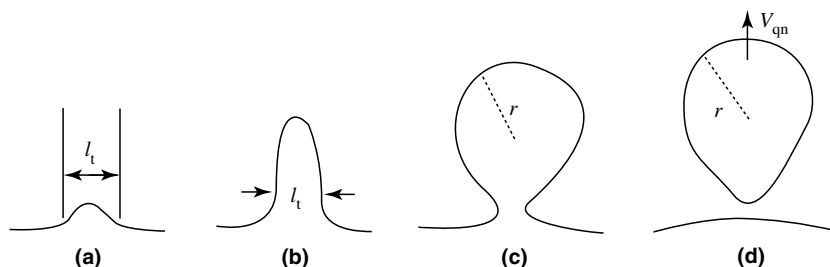

Fig. 1. Four steps to eject a quark nugget during the formation of a quark star.

What kind of quark nugget could be ejected? Let us compare the energies below

$$\frac{E_t}{E_g} \sim \frac{R^2 v_t^2}{GMr} \simeq 8 \times 10^{-15} R_6^{-1} \frac{v_t^2}{r},$$

$$\frac{E_t}{E_s} \sim \frac{\rho_0 v_t^2 r}{\gamma} \simeq 2 \times 10^{-8} \gamma_{100}^{-1} v_t^2 r, \tag{2}$$

where $\rho_0 \simeq 4 \times 10^4$ g/cm^3 is the average density of quark stars with mass $< \sim M_\odot$, $R = R_6 \times 10^6$ cm, $\gamma = \gamma_{100} \times 100$ MeV fm^{-2}. Both E_t/E_g and E_t/E_s should be much larger than 1 if quark nuggets are ejected. We conclude then from Eq. (2) that: (i) it needs $v_t > 10^8$ cm/s that nuggets with mass $> 10^{20}$ g can be ejected; (ii) nuggets with higher mass could be ejected from nascent quark stars with lower mass; (iii) ejection of nuggets with planet masses (e.g., the Earth's mass $\sim 10^{27}$ g) could be possible if $v_t > 10^9$ cm/s; (iv) a nascent quark star with weak turbulent (e.g., $v_t < 10^4$ cm/s) could hardly evaporate nuggets with low mass (e.g., $< 10^{14}$ g). Hadrons may not evaporate from the surface of a quark star with temperature being much lower than 100 MeV if γ is order of 100 MeV fm^{-2}. The unknown parameter v_t would be probably much larger than 10^7 cm/s since observations show that pulsar-like stars receive a large kick velocity (of order a few hundred to a thousand km/s) at birth.

These ejecta could be captured by the center strange star (with mass M and radius R) for v_t should be smaller than the escape velocity $\sqrt{2GM/R} \sim 10^{10}$ cm/s. We then conclude that *quark planets* (i.e., planets of quark matter, with possible mass from much lower than earth mass to about a Jupiter mass) would form simultaneously during the formation of a strange star with strong turbulence if the surface energy is reasonable ($\gamma < 10^3$ MeV fm^{-3}). This consequence could be tested by searching quark planets around pulsars. This is also an alternative mechanism for creating pulsar-planet systems, the first one of which was discovered by Wolszczan and Frail [35].

2.2. Global oscillation of quark stars

A spherical fluid should be deform by rotation. For a quark star of incompressible fluid with axis-symmetric deformation, the star's surface can be described as

$$r(\theta) = \lambda R \left[1 - \sum_{i=2}^{n} \alpha_i P_i(\cos\theta) \right], \tag{3}$$

where the parameter λ is determined by a constant volume of $4\pi R^3/3$. In case of $\alpha_2 \ll 1$ (and thus terms with orders being higher than α_2^2 are neglected), the quadrupole deformation ($i = 2$), which will only be noted as following, is actually of an ellipsoidal figure with an ellipticity $\varepsilon = \alpha_2$ and axes $a = R/\sqrt{1-\varepsilon} = b > c = R(1-\varepsilon)$.

In case of a rotating fluid star with $T/|W| \ll 0.1375$, the ellipticity $\varepsilon \equiv (I - I_0)/I_0$ and the eccentricity $e = 1 - c^2/a^2$ are related by $\varepsilon \approx e^2/3$, where $I(\Omega)$, as a function of the spin frequency Ω, is the total moment of inertia, with $I_0 =$

$2MR^2/5$ for stars with uniform density. The total energy, E, of a rotating fluid star is then the sum of gravitation, rotation, volume, and surface energies. A calculation similar to the nuclear liquid drop model[2] [15] shows

$$E = E_{\text{gravi}} + E_v + E_s + E_{\text{rot}}$$

$$= E_0 + (A_g + A_s)\varepsilon^2 + \frac{L^2}{2I_0} \frac{1}{(1+\varepsilon)}, \tag{4}$$

where E_0 is the energy for a non-rotating spherical star, $L = I\Omega$ the star's angular momentum, $I(\varepsilon)$ the momentum of inertia, and the coefficients A_g and A_s measure the increases of gravitation and surface energies, respectively, of the star,

$$A_g = \frac{3}{25} \frac{GM^2}{R}, \quad A_s = \frac{8\pi}{5} \gamma R^2. \tag{5}$$

The surface energy contribution to the deformation should be negligible since the parameter

$$\eta \equiv \frac{A_s}{A_g} \simeq \frac{15}{2\pi G \rho_0^2} \cdot \frac{\gamma}{R^3} \sim 10^{-18} \gamma_{100} R_6^{-3} \tag{6}$$

is much smaller than 1 unless the star is very low-massive (i.e., mass $< \sim 10^{14}$ g for strangelets) and/or $\gamma \gg 100$ MeV fm^{-2}.

By using $dE(\varepsilon)/d\varepsilon = 0$ with Eq. (4), neglecting A_s, one comes to the rotating figures of conventional Maclaurin spheroids,[3]

$$\Omega \simeq 2\sqrt{\frac{2\pi G \rho_0}{15}} e_0, \text{ or,}$$

$$\varepsilon_0(P) \simeq \frac{5\Omega^2}{8\pi G \rho_0} \simeq 3 \times 10^{-3} P_{10\,\text{ms}}^{-2}, \tag{7}$$

where e_0 and ε_0 are the eccentricity and the ellipticity, respectively, of a fluid star at equilibrium, which are only a function of spin, provided that ρ_0 is a constant, the spin period $P = 2\pi/\Omega = P_{10\,\text{ms}} \times 10$ ms. One can also have

$$\frac{d^2 E(\varepsilon)}{d\varepsilon^2} = 2(A_g + A_s) + \frac{L^2}{I_0(1+\varepsilon)^3} > 0, \tag{8}$$

which means the equilibrium state of $\varepsilon = \varepsilon_0$ is stable. We expect that a fluid quark star, either at birth or in its lifetime, should *oscillate* globally around ε_0, though the oscillation frequency and amplitude are still not computed now. If the amplitude is large, this quadrupole oscillation could result in enough gravitational radiation to be detected by future advanced facilities.

Additionally, this oscillation modes may help us to distinguish fluid and solid quark star models. A protoquark star should be in a fluid state, but the star may solidified soon after its birth [36,39]. Therefore, we should find ways to differentiate solid and fluid quark stars in their later time. Due to the strong shear force, a solid quark star

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[2] The Coulomb interaction there is replaced by gravitational interaction, with a sign-change.

[3] Note that the approximation of $\varepsilon_0(P)$ can only be valid for $P_{1\,\text{ms}} \gg 1$.

may oscillate with much smaller amplitude but much higher frequency than that of a fluid quark star. Besides detecting the gravitational wave, pulsar timing in radio, optical, as well as X-ray bands could be effective if the star has pulsed emission, because the rotation period changes during the oscillation.

Could the oscillation amplitude be larger enough to fragmentate a fluid quark star? Unlike the case of nuclear fission, where the Coulomb interaction favors, the gravitational interaction prevent a quark star to fragmentate. It needs then energy to "excite" the star, with order of $E_{ex} \sim 0.6G(M/2)^2/R$. If pulsar kick [28], with energy of $E_{ki} \sim 0.5MV_k^2$, plays a major role to fission a quark star, we note that the kick velocity V_k should be greater than $\sim R\sqrt{2\pi G\rho_0/5} \simeq 6 \times 10^3 R$ cm/s. Therefore, a quark star with large kick (or turbulent) energy and low-mass could fragmentate.

3. Gravitational waves due to r-mode instability

In Newtonian theory, a rapidly rotating fluid Maclaurin spheroid is secularly unstable to become a Jacobi spheroid, which is non-axisymmetric, if the ratio of the rotational kinetic energy to the absolute value of the gravitational potential energy $T/|W| > 0.1375$. In the general relativistic case, as was shown by Chandrasekhar [10] and Friedman and Schutz [13] gravitational radiation reaction amplifies an oscillation mode, and it is then found that the critical value of $T/|W|$ for the onset of the instability could be much smaller than 0.1375 for neutron stars with mass $>\sim M_\odot$. This sort of non-axisymmetric stellar oscillations will inevitably result in gravitational wave radiation.

A kind of oscillation mode, so-called r-mode, is focused on in the literatures since the work by Andersson [3], Friedman and Morsink [12] and Lindblom et al. [22]. The r-mode oscillation is also called as the Rossby waves that are observed in the Earth's ocean and atmosphere, the restoring force of which is the Coriolis force. This instability may increase forever if no dissipation occurs. Therefore, whether the instability can appear and how much the oscillation amplitude is depend on the interior structure of pulsars, which is a tremendously complicated issue in supranuclear physics.

To make sense of the generic nature for r-mode instability in different star-modes but avoiding an uncertainty in microphysics, we just present a rough calculation below. The critical angular frequency, Ω, limited by the gravitational radiation due to r-mode instability, are determined by comparing the damping/growth times due to the various mechanisms for energy dissipation in a given star-model,

$$\frac{1}{\tau_{gw}} + \frac{1}{\tau_{sv}} + \frac{1}{\tau_{bv}} = 0, \tag{9}$$

where the timescales for the instability are estimated to be [24],

$$\begin{cases} \tau_{gw} = -3.85 \times 10^{81}\Omega^{-6}M^{-1}R^{-4}, \\ \tau_{sv}^{ns} = 1.01 \times 10^{-7}M^{-1}R^5T^2, \\ \tau_{bv}^{ns} = 1.29 \times 10^{61}\Omega^2M^{-1}R^5T^{-6}, \\ \tau_{sv}^{ss} = 1.85 \times 10^{-9}\alpha_s^{5/3}M^{-5/9}R^{11/3}T^{5/3}, \\ \tau_{bv}^{ss} = 5.75 \times 10^{-2}m_{100}^{-4}\Omega^2R^2T^{-2}, \end{cases} \tag{10}$$

where superscript "ns" ("ss") denotes neutron (bare strange) star model, τ_{sv} and τ_{bv} are the dissipation timescales due to shear and bulk viscosities, respectively, and α_s the coupling constant of strong interaction, T the temperature, m_{100} the strange quark mass in 100 MeV. It is assumed in Eq. (10) that the modified URCA process dominate the weak interaction in normal neutron stars.

Strange stars could have very low-masses, even of $10^{-3}M_\odot$ (a strange star with planet mass could also be called as a strange *quark* planet). A low-mass normal neutron star could not be likely because of (i) the minimum mass of a stable neutron star is $\sim 0.1M_\odot$ [30], and (ii) the gravitation-released energy of possible low-mass neutron star during an iron-core collapse supernova, $E_g \sim GM^2/R$, should be much smaller than $\sim 10^{53}$ erg due to the relation $M \sim R^{-3}$ for low-mass neutron stars, but it seems not successful in modern supernova simulations even in case of $E_g \sim 10^{53}$ erg [9,21]. Therefore, in the following calculations, we consider low-mass issues for bare strange stars, but apply only canonical mass of $1.4M_\odot$ for normal neutron stars.

No r-model instability occurs in a solid star since the Coriolis-restoration force does not work here. The break frequency of low-mass bare strange stars could be approximately a constant,

$$\Omega_0 = \sqrt{\frac{GM}{R^3}} = 1.1 \times 10^4 \text{ s}^{-1} \tag{11}$$

with a prefactor of ~ 0.65 at most for $M \sim M_\odot$ and $R \sim 10^6$ cm [14], where the bag constant $B = 60$ MeV fm^{-3}. Let us express the critical frequency in unit of Ω_0, through calculations based on Eq. (9). It is found in Fig. 2 that: (i) gravitational wave radiates more likely from proto-neutron stars than from proto-strange stars; (ii) the r-mode instability could not occur in fluid bare strange stars with radii being smaller than ~ 5 km (or mass of a few $0.1M_\odot$) unless these stars rotates faster than the break frequency and (iii) the above conclusions do not change significantly in the reasonable parameter-space of B, α_s, and m_s. To estimate strength of gravitational waves from this oscillation, one needs to simulate the non-linear increase of the instability, which is not certain yet.

Some recent observations in X-ray astronomy could hint the existence of low-mass bare strange stars [38]. The radiation radii (of, e.g., 1E 1207.4-5209 and RX J1856.5-3754) are only a few kilometers. No gravitational wave emission could be detected from such fluid stars even they spin only with a period of ~ 1 ms.
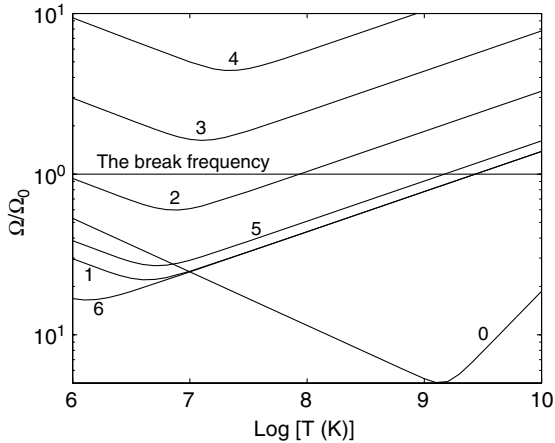
Fig. 2. Temperature dependence of the critical angular frequency, in unit of Ω_0, due to r-mode instability for normal neutron stars and bare strange stars. The critical lines are numbered from "0" to "6". "0": normal neutron stars; "1": bare strange stars (BSSs) with mass $M = M_\odot$, the bag constant $B = 60$ MeV fm^{-3}, the strange quark mass $m_s = 100$ MeV, and the coupling constant $\alpha_s = 0.1$; "2": BSSs with $M = 0.1M_\odot$, $B = 60$ MeV fm^{-3}, $m_s = 100$ MeV, and $\alpha_s = 0.01$; "3": BSSs with $M = 0.1M_\odot$, $B = 60$ MeV fm^{-3}, $m_s = 100$ MeV, and $\alpha_s = 0.1$; "4": BSSs with $M = 0.001M_\odot$, $B = 60$ MeV fm^{-3}, $m_s = 100$ MeV, and $\alpha_s = 0.1$; "5": BSSs with $M = M_\odot$, $B = 110$ MeV fm^{-3}, $m_s = 100$ MeV, and $\alpha_s = 0.1$; "6": BSSs with $M = M_\odot$, $B = 60$ MeV fm^{-3}, $m_s = 100$ MeV, and $\alpha_s = 0.9$.

4. Gravitational waves from wobbling stars

Gravitational wave radiation from pulsars could be classified as (i) emission due to the normal modes of oscillation of fluid matter (e.g., the r-mode instability discussed in Section 3), and (ii) emission due to solid deforming which will be focused in this section. Protoquark stars should be in a fluid state when their temperatures are order of 10 MeV, but would be solidified as they cool to very low temperatures [36]. Assuming the ellipticity of a solid quark star, with an initial spin period P_0, keeps the same as that of the star just in its fluid phase, we expect realistic ellipticity ϵ of the star in the solid state satisfies

$$\varepsilon_0(P_0) > \epsilon > \varepsilon_0(P) \tag{12}$$

as the star spins down to a period of P, due to the shear force that prevent the star to deform. For the Earth which is spinning down, the mean density is 5.5 g/cm^3, the ellipticity observed $\epsilon = 0.00223$, one has $\varepsilon_0(24\,\text{h}) = 0.00287 > \sim \epsilon$ from Eq. (7). Additionally, a free precession mode called the Chandler wobble, which was formulated and expected by Leonhard Euler, with a period of 435 days suggests a good approximation of ϵ too. This means that the suggestion of Eq. (12) is at least workable for the equilibrium figure of the Earth.[4]

However, strain energy develops when a solid quark star spins down, which is proportional to $[\varepsilon_0(P_0) - \epsilon]^2$. The first

glitch of the quark star occurs when the strain energy reaches a critical value [43]. Subsequent glitches take place too as the stellar stress increases to critical points. We note that the star's ellipticity decreases after glitches due to (i) the release of strain energy and (ii) possible increase of mean density if stellar volume shrinks as the star cools.

A star's ellipticity could be observed if the star's free precession mode is discovered due to the relation $\epsilon \simeq P/P_{\text{prece}}$, where P_{prece} is the precession period. Evidence for free precession for PSR B1828-11 was provided by Stairs et al. [31]: periodic timing variation and changes in pulse duration. The derived ellipticity is $\sim P/P_{\text{prece}} \simeq 0.405$ s/500 d $\simeq 10^{-8}$ *if* the precession is really in a free mode. But, based on Eq. (7), the ellipticity is $\varepsilon_0(405\,\text{ms}) \simeq 2 \times 10^{-6}$, which is 2 orders larger than that expected. This conclusion conflicts with Eq. (12). Why is PSR B1828-11 so "round"? An answer proposed here is that this observed precession mode could actually be torqued by external force, rather than free. In fact, a disk-forced precession model for PSR B1828-11 was provided by Qiao et al. [29], but other source torques by, e.g., magnetodipole radiation or planet(s), are also possible. The precession period could be very long (and thus results in a much small ellipticity if assuming free precession) if torques are not strong. Due to the torque exerted by the Moon and the Sun, the spinning Earth shows a forced precession period of 25,700 yr, being much larger than 435 d, the period of free precession. By the way, planet-torque induced precession modes could be effective to search planets around pulsars. Nevertheless, a free precession mode could have been discovered in a bursting radio source, GCRT J1745−3009, in the direction of the Galactic center region [42]. The ellipticity derived from P/P_{prece} is $\sim 10/77 \simeq 2 \times 10^{-6}P_{10\,\text{ms}}$. Based on Eqs. (7) and (12), the spin period should be $P > 0.1$ s, which is very reasonable for normal pulsars. If the pulsar's ellipticity tracks almost $\varepsilon_0(P)$ due to glitches, $\varepsilon > \sim \varepsilon_0(P)$, then its ellipticity $\epsilon > \sim 2 \times 10^{-5}$ and period $P > \sim 0.1$ s. Statistically, a pulsar with $P > \sim 0.1$ s could be a radio nulling (even extremely, with null fraction >90%) pulsar [8], which fits the model provided by Zhu and Xu [42].

A pulsar must be non-axisymmetric in order to radiate gravitationally. A wobbling pulsar, either freely or forcedly, may thus radiation gravitational waves. This wave results in a perturbed metric $h_{\mu\nu}$ of space-time ($g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$), which is order of h_0 being given by Jaranowski et al. [18] and Jones and Andersson [19],

$$h_0 = \frac{128\pi^3 G\rho_0}{15c^4} \cdot \frac{\epsilon R^5}{\text{d}P^2}\theta \approx 2.8 \times 10^{-20}\epsilon R_6^5 d_{\text{kpc}}^{-1}P_{10\,\text{ms}}^{-2}\theta, \tag{13}$$

where approximations $I \simeq 0.4MR^2$ and $M \simeq 4\pi R^3\rho_0/3$ are applied for solid quark stars in the right equation, the pulsar's distance to earth is $d = d_{\text{kpc}} \times 1$ kpc, θ is the wobble angle. LIGO is sensitive to height frequency waves, which recently puts upper limits on h_0 for 28 known pulsars through the second LIGO science run [1]. The upper limits are order of 10^{-24}, which means approximately an limit of

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[4] Among solid astrophysical objects, only the Earth is investigated mostly extensively. One has then to learn from solid geophysicists in order to develop a comprehensive astrophysical model for solid quark stars.

Table 1
The upper limits of the radii and masses of 28 pulsars targeted in the second science run of LIGO, which depend on the wobble angle θ

Pulsar name	Period P (ms)	Distance d (kpc)	h_0 (10^{-24})	Radius·$\theta^{1/5}$ (km)	Mass·$\theta^{3/5}$ ($M_\odot$)
B0531+21*	33.08	2.00	41	19.6	6.4
J1913+1011*	35.91	4.48	51	18.6	5.5
B1951+32*	39.53	2.50	48	22.4	9.4
B0021−72C	5.76	4.80	4.3	2.6	0.015
B0021−72D	5.36	4.80	4.1	2.4	0.012
B0021−72F	2.62	4.80	7.2	1.5	0.0030
B0021−72G	4.04	4.80	4.1	1.9	0.0061
B0021−72L	4.35	4.80	2.9	1.9	0.0059
B0021−72M	3.68	4.80	3.3	1.7	0.0043
B0021−72N	3.05	4.80	4.0	1.5	0.0031
J0030+0451	4.87	0.23	3.8	4.1	0.056
J0711−6830	5.49	1.04	2.4	3.0	0.023
J1024−0719	5.16	0.35	3.9	3.9	0.051
B1516+02A	5.55	7.80	3.6	2.2	0.0090
J1629−6902	6.00	1.36	2.3	3.0	0.024
J1721−2457	3.50	1.56	4.0	2.1	0.0084
J1730−2304	8.12	0.51	3.1	5.0	0.11
J1744−1134	4.07	0.36	5.9	3.5	0.037
J1748−2446C	8.44	8.70	3.1	2.9	0.021
B1820−30A	5.44	7.90	4.2	2.2	0.0094
B1821−24	3.05	4.90	5.6	1.6	0.0037
J1910−5959B	8.36	4.00	2.4	3.2	0.028
J1910−5959C	5.28	4.00	3.3	2.4	0.011
J1910−5959D	9.04	4.00	1.7	3.2	0.028
J1910−5959E	4.57	4.00	7.5	2.5	0.013
J1939+2134	1.56	3.60	13	1.2	0.0015
J2124−3358	4.93	0.25	3.1	3.9	0.048
J2322+2057	4.81	0.78	4.1	3.2	0.027

Normal pulsars listed are starred (*), others are millisecond pulsars. The h_0 limits are from Abbott et al. [1], while the pulsar data are obtained through http://www.atnf.csiro.au/research/pulsar/psrcat/.

$\epsilon R_6^5 < 10^{-4}$. This observation may not conflict with Eqs. (7) and (12) if the pulsars' radii are slightly smaller than 10 km. In fact, possible candidates for low-mass quark stars with small radii are proposed by Xu [38]. Only three normal pulsars are listed in Table 1 of [1]: the Crab (B0531+21), J1913+1011, and B1951+32, others are millisecond pulsars.

It is conventionally suggested that millisecond pulsars are recycled, with a spinup history during binary accretion, and the ellipticity may not follow Eq. (12) since $\varepsilon_0(P) > \epsilon$ for such stars during their spinup phases. Nonetheless, since millisecond pulsars are now in a spindown phase, we may simply suggest $\varepsilon_0(P) \simeq \epsilon$, which is valid for the Earth, and calculate the upper limits of pulsars' radii and thus masses according to Eqs. (7) and (13). Note that Eq. (12) is still effective if millisecond pulsars are born during an AIC (accretion-induced collapse of white dwarfs) process [38]. The upper limits of pulsars' radii and masses are calculated, which are presented in Table 1. Large radius limits (>10 km and thus high mass limit, $>M_\odot$) are for the three normal pulsars, which does not show any constrain on the equation of state since pulsar's maximum mass is only $\sim 2M_\odot$. However, the LIGOs observation suggests a small limit of $R \cdot \theta^{1/5}$ (a few kilometers), and thus a small limit of $M \cdot \theta^{3/5}$ (between $10^{-2} M_\odot$ and $10^{-3} M_\odot$), for those millisecond pulsars. This hints either a low-mass ($\ll M_\odot$) of pulsars or a small wobble angle θ. Factually, in order to explain its polarization behavior of radio pulses and the integrated profile (pulse widths of main-pulse and inter-pulse, and the separation between them), the fastest[5] rotating millisecond pulsar PSR J1939+2134 (B1937+21) is supposed to have mass $<0.2M_\odot$ and radius <1 km [41]. This conclusion does not conflict with the value in Table 1. Observationally, the precession angle (i.e., the angle between the spin vector and total angular momentum), which is smaller than θ, should be smaller than the pulsar radio beam angle (a few tens of degrees). The radius upper limits for the millisecond pulsars could be about two times the values listed in Table 1, and then the mass limit of (3–10) times, if $\theta \simeq (1$–$10°)$. Actually, a small radius of pulsar-like stars is not surprising since we have detected six central compact objects in supernova remnants (CCOs) with black-body radius of 0.3–2.4 km [27], and seven Dim Thermal Neutron stars (DTNs) with possible radius of a few kilometers [16].

4.1. Elliptic deformation vs. bumpy distortion

The lack of symmetry about a pulsar's rotation axis could be the result of either elliptic deformation because of rotation, which is discussed in Sections 2 and 4. or

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[5] A 1.39 ms radio pulsar could have been discovered in the globular cluster Terzan 5 by the Robert C. Green Bank Telescope (GBT).

bumpy distortion (i.e., localized mountains on stellar surface). Strong magnetic field is suggested for producing the bumps, since the magnetic and rotating axes are generally not aligned. However for quark stars, this mechanisms may not work due to a relatively negligible magnetic force, $(B^2/8\pi)/(\rho_0 c^2) \sim 10^{-5} B_{16}$, even for fields $(B = B_{16} \times 10^{16}$ G) as strong as 10^{16} G. Glitches of solid quark stars could produce bumps, with a maximum ellipticity [26],

$$\epsilon_{\max} \sim 10^{-3} \left(\frac{\sigma_{\max}}{10^{-2}} \right) R_6^{-6} (1 + 0.084 R_6^2)^{-1}, \tag{14}$$

where $\sigma_{\max}$ is the stellar break strain. This ellipticity is larger for low-mass quark stars due to weaker gravity. But for normal neutron stars, Owen [26] derived the maximum elastic deformation, $\epsilon_{\max}$, induced from shear stresses is typically only 6.0×10^{-7}. The real ellipticity of a quark star could be far from $\epsilon_{\max}$, but may be approximately $\varepsilon_0(P)$, due to stress releases through star-quake induced glitches [43].

5. Conclusions

Detection of gravitational waves can certainly test the general theory of relativity, and also open a new window for us to observe astrophysical phenomena. Observations in gravitational wave band may reveal the nature of pulsar-like stars, and help to answer numerous questions: Are pulsar-like stars normal neutron stars or quark stars? Is quark matter with height density but low temperature in a solid state? Do quark stars with low-masses exist in the Universe? The paper is summarized as follows:

(1) During the birth of quark stars, quark nuggets may be ejected, and the global quadrupole oscillation around ellipticity $\varepsilon_0(P)$ (see Eq. (7)) could also be a source for gravitational waves.
(2) No gravitational wave originated from *r*-mode instability could be detected for pulsars to be quark stars with low-masses or in a solid state, for isolated as well as accreting compact stars, even the compact stars spin at sub-millisecond periods.
(3) Through the second LIGO science run, we may find upper limits of masses and radii for millisecond pulsars, although one can not constrain on the masses and radii of normal neutron stars. The radius of PSR B1937+21 could be smaller than $\sim$2 km if its wobble angle θ is between 1° and 10° Future LIGO observations could put tight constraint on $R \cdot \theta^{1/5}$ and thus $M \cdot \theta^{3/5}$ if pulsar-like stars are actually solid quark stars.
(4) We suggest the precession mode discovered in PSR B1828−11 is not free, but could be forced by fossil disk or planets. The geometrical parameters of torqued precession might be obtained by precise timing of radio pulses. The bursting radio source, GCRT J1745−3009, could be a freely processing radio pulsar with spin period $\sim$0.1 s, which could be tested by searching pulsed emission from the source.

As the mass–radius relation for low-mass quark stars ($M \propto R^3$) is in striking contrast to those of normal neutron stars with low-masses ($M \propto R^{-3}$), gravitational radiation from low-mass neutron stars and quark stars should be very different for processes of both fluid instability and stellar wobbling. We have just studied mass-dependent waves for quark stars, but it is also necessary to obtain the mass-dependent behavior for normal neutron stars, in order to put theoretical models directly in front of gravitational wave observations. Additionally, similar to the case of double white dwarf binaries [32], detecting gravitational wave via LISA (the Laser Interferometer Space Antenna) from binary quark stars is also valuable to constrain the masses and radius. The radiation reaction, tides, and mass transfer are very different for binary low-mass quark stars, white dwarfs, and normal neutron stars.

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