Chinese Physics Letters

The Fastest Rotating Pulsar: a Strange Star?

The Fastest Rotating Pulsar: a Strange Star? *

XU Ren-Xin(徐仁新), XU Xuan-Bin(徐轩彬), WU Xin-Ji(吴鑫基)
CAS-PKU Joint Beijing Astrophysical Center and Department of Astronomy, Peking University, Beijing 100871

(Received 3 January 2001)

According to the observational limits on the radius and mass, the fastest rotating pulsar (PSR 1937+21) is probably a strange star, or at least some neutron star equations of state should be ruled out, if we suggest that a dipole magnetic field is relevant to its radio emission. We presume that the millisecond pulsar is a strange star with much *low mass, small radius and weak magnetic moment.*

PACS: 97.60.G, 97.60.J, 95.30.C

Pulsars are conventionally modelled as neutron stars, but a strange star[1,2] model for pulsars is also proposed (see also a review, for example, by Xu *et al.*[3]). Therefore it is of great importance to find observationally unambiguous features to distinguish strange stars from neutron stars. There are three hopeful ways known hitherto to identify a strange star, which are based upon the differences of viscosity, mass–radius relation and the surface condition between neutron and strange stars.[4] Recently, Kapoor and Shukre[5] suggested the constraint of the equations of state of neutron stars by Rankin's experiential line[6] of the core emission of radio pulsars, and found the restriction that the pulsar masses $M \leq 2.5 M_\odot$ and radii $R \leq 10.5$ km, indicating that puslars are strange stars. However, Kapoor and Shukre's work[5] has at least two unseemly points: (1) the polar cap is defined for aligned rotators (rather than for orthogonal rotators) in their calculation; (2) the Rankin line is doubtless untrue due to many observational and statistical uncertainties. By redefining the polar cap for orthogonal rotators, still we cannot obtain a conclusion that puslars are strange stars, or that some neutron star equations of state are ruled out, if the Rankin line is used to constrain pulsar masses and radii. Nevertheless, when applying this method to the fastest rotating pulsar, PSR 1937+21, we conclude that the pulsar may be a strange star, or at least some equations of state should be ruled out. In order to find solid evidence to shown that PSR 1937+21 is a strange star, it is strongly suggested to measure precisely the maximum rate of position-angle swing by further polarization observations.

Now, we improve Kapoor and Shukre's computation by removing the first unseemliness. In a spherical coordinate system with magnetic axis μ being chosen as the z-axis, one has the following form for a position vector $\boldsymbol{r}$ in a dipole field line denoted by parameter λ,[7]

$$r = \lambda \frac{cP}{2\pi} \sin^2 \theta, \tag{1}$$

where θ is the polar angle, P the rotation period, c the speed of light, and λ characterizes the sort of field lines. Based on Eq. (1), the angle θ_μ between $\boldsymbol{\mu}$ and the direction of magnetic field at $\boldsymbol{r}$ reads,

$$\cos \theta_\mu = \frac{2\lambda - 3\mathcal{R}}{\sqrt{\lambda(4\lambda - 3\mathcal{R})}}, \tag{2}$$

where $\mathcal{R} \equiv 2\pi r/(cP) \ll 1$ for points near pulsar surface. Expanding function θ_μ in the vicinity of $\mathcal{R} = 0$, one comes to

$$\theta_\mu = \frac{3}{2\sqrt{\lambda}} \mathcal{R}^{1/2} + \frac{3}{8\lambda\sqrt{\lambda}} \mathcal{R}^{3/2} + O(\mathcal{R}^{5/2}). \tag{3}$$

Assuming $\lambda = \lambda_0$ for the last-open-field lines (note that λ_0 is a function of inclination angle α), one can obtain the beam radius ρ for the radiation at an emission height $h = r - R$ (R is the radius of pulsar), neglecting the terms being equal to or higher than $\mathcal{R}^{3/2}$ in Eq. (3),

$$\rho = \frac{3}{2} \sqrt{\frac{2\pi}{\lambda_0 c}} \frac{r^{1/2}}{P^{1/2}} = \frac{1.24^\circ}{\sqrt{\lambda_0 P}} \sqrt{\frac{r}{10\mathrm{km}}}. \tag{4}$$

For core emissions ($r \gtrsim R$), which are supposed to originate from regions near pulsar surfaces,[6] general relativistic effects are not negligible due to the space-time curvature.[8] Two such effects, squeezing of the dipole magnetic field and bending of the radio wave, can be represented approximately by two factors,[8,5] $f_{\rm sqz}$ and $f_{\rm bnd}$, respectively, so that the beam radius ρ can be re-written as

$$\rho = \frac{3}{2} \sqrt{\frac{2\pi}{\lambda_0 c}} \frac{r^{1/2}}{P^{1/2}} = \frac{1.24^\circ}{\sqrt{\lambda_0 P}} \sqrt{\frac{r}{10\mathrm{km}}} f_{\rm sqz} f_{\rm bnd}, \tag{5}$$

where

$$f_{\rm sqz} = \left(1 + \frac{3GM}{2c^2 r}\right)^{-1/2},$$

$$f_{\rm bnd} = \frac{1}{3}\left[2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1/2}\right],$$

G is the gravitation constant, M the pulsar mass.

---

*Supported by the National Natural Science Foundation of China under Grant No. 19803001, and the Special Funds for Major State Basic Research Projects of China.

We are to calculate the parameter λ_0 of last-open-field lines below, assuming that only the plasma within the light cylinder with radius $r_{\rm lc} = cP/(2\pi)$ can corotate with the star, i.e. field lines that penetrate beyond the light cylinder should be open. Generally, λ_0 is a function of inclination angle α. Nevertheless, one can obtain simple formulae of λ_0 in two special cases. Obviously, $\lambda_0(\alpha = 0) = 1$ for an aligned rotator according to Eq. (1). Another formula of λ_0 can be easily reached for an orthogonal rotator ($\alpha = 90°$). The conal angle θ_0 of the null surface, where the magnetic fields are perpendicular to magnetic axis μ, can be obtained from Eq. (2) by setting $\theta_\mu = 90°$,

$$\theta_0 = \sin^{-1}\sqrt{\frac{2}{3}}. \tag{6}$$

Let $r\cos\theta_0 = r_{\rm lc}$, one finds

$$\lambda_0(\alpha = 90°) = \frac{3\sqrt{3}}{2}, \tag{7}$$

which is 2.6 times of $\lambda_0(\alpha = 0°)$.

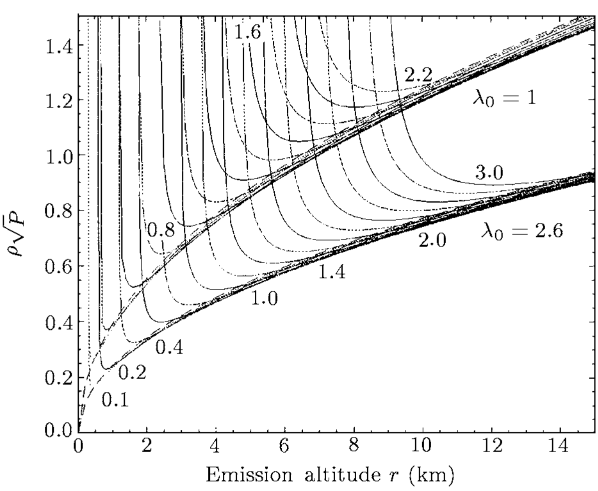

Fig. 1. Calculated results based on Eq.(5) show the variations of $\rho \times \sqrt{P}$ (in deg$\cdot$s$^{1/2}$) as a function of emission altitude r for different pulsar masses ranging from $0.1M_\odot$ to $\gtrsim 2M_\odot$. We present results of $\lambda_0 = 1$ (aligned rotators) and $\lambda_0 = 3\sqrt{3}/2$ (orthogonal rotators) for two limits; an actual value for a pulsar with artificial inclination angle α should be between those two corresponding values. The dashed lines are calculated by Eq. (4), which are not relevant to pulsar mass.

Following the representation by Kapoor and Shukre,[5] we calculate the value, $\rho \times \sqrt{P}$, as a function of emission altitude r for pulsars with core emission according to Eq. (5). The calculated results are shown in Fig. 1. Contrary to the result of Kapoor and Shukre,[5] it is clear from Fig. 1 that the Rankin line (i.e. $\rho \times \sqrt{P} = 1.225°$ s$^{1/2}$) cannot effectively put any limit on the equations of state of neutron stars or conduce toward any indication that pulsars are strange stars if we assume that Rankin's samples are of nearly orthogonal rotators. Nevertheless, we may still obtain some valuable conclusions from above calculation for some special pulsars, for instance, the fastest rotating millisecond pulsar, PSR 1937+21.

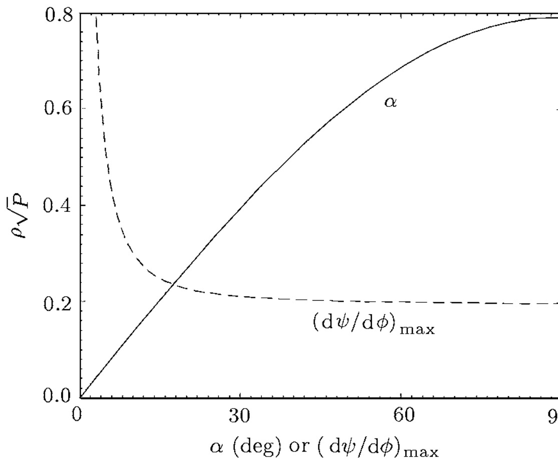

Fig. 2. Beam angular radius times square root of period ($\rho\sqrt{P}$) of PSR 1937+21 versus possible different inclination angles α for the case of $({\rm d}\psi/{\rm d}\phi)_{\rm max} = 3$ (dashed line), or versus maximum rates of position-angle swing $({\rm d}\psi/{\rm d}\phi)_{\rm max}$ for $\alpha = 90°$ (solid line).

The rotation period of PSR 1937+21 is $P = 1.558$ ms, the smallest pulsar observed, and the rate of period change is $\dot{P} = 10^{-19}$ s $\cdot$ s^{-1}. The radius of the light cylinder of the star is $r_{\rm lc} = cP/(2\pi) = 74$ km. The corotation radius, defined by the balance of gravitational force and the centrifugal force, is $r_{\rm c} = [GM/(4\pi^2)]^{1/3}P^{2/3} = 20(M/M_\odot)^{1/3}$km, where M is the mass of PSR 1937+21. Therefore, the pulsar radius R satisfies

$$R < \min(r_{\rm lc}, r_{\rm c}) = 20(M/M_\odot)^{1/3}\ {\rm km}. \tag{8}$$

Another circumscription of the stellar radius R and the mass M may arise from the inclusion of the general relativistic effect.[5] The pulse width $\Delta\phi$ of PSR 1937+21 is about 10° at 600 MHz and 1.4 GHz (EPN database), which is difficult to be explained geometrically if we suggest PSR 1937+21 has a radius of 10 km for canonical pulsars.[9] Thus, Gil proposed a model[10] in which a quadrupole magnetic field geometry is assumed. However, an alternative conjecture to overcome the difficulty is that PSR 1937+21 may have unusual small radius. Taking the simplest proposal that $\alpha = 90°$ and impact angle $\beta = 0$, we know $\Delta\phi = 2\rho$, and therefore $\rho\sqrt{P} = 0.2°$ s$^{1/2}$. According to the results in Fig. 1, it is found that

$$M < 0.2\ M_\odot \quad \text{and} \quad R < 1\ \text{km}. \tag{9}$$

These stringent limits have to result in the conclusion that PSR 1937+21 is a strange star rather than a neutron star because of the strikingly different mass–radius relations.

However, the conclusion in Eq. (9) is not so solid. In fact, there are two models proposed to account for the inter-pulse emission:[11] the single-pole model and the double-pole model. Three observational facts[10] favour a double-pole model of inter-pulse origin of PSR 1937+21. (1) The gradient of the position angle at inter-pulse centre has the same value as that at the main-pulse centre. (2) The separation of the main-pulse and the inter-pulse is nearly 180°. (3) The intensities of the main-pulse and the inter-pulse are roughly equal. The observed maximum rate of position angle swing at the main-pulse centre is $(\mathrm{d}\psi/\mathrm{d}\phi)_{\max} \approx 3$. In the standard rotating vector model, the impact angle β should be

$$\beta = \sin^{-1}\left[\frac{\sin\alpha}{(\mathrm{d}\psi/\mathrm{d}\phi)_{\max}}\right] \sim 19.5^\circ, \tag{10}$$

and the beam radius ρ thus can be derived as

$$\rho = \cos^{-1}\left[\cos\beta - 2\sin\alpha\sin(\alpha+\beta)\sin^2\frac{\Delta\phi}{4}\right] \sim 20.1^\circ, \tag{11}$$

if we take $\alpha = 90^\circ$, $(\mathrm{d}\psi/\mathrm{d}\phi)_{\max} = 3$ and $\Delta\phi = 10^\circ$. In this case, we find $\rho\sqrt{P} = 0.79^\circ\ \mathrm{s}^{1/2}$, and the limits thus are

$$M < 2.4\ M_\odot \quad \text{and} \quad R < 11.5\ \text{km} \tag{12}$$

based on Fig. 1.

Table 1. Total gravitational mass–energy M and the circumferential radii R at the equator of rotating ($P = 1.558$ ms) neutron stars with maximum mass for five kinds of equations of state.

EOS*	$M/M_\odot$	R(km)
A (Reid soft core)	1.6604	8.82
L (Mean field)	2.7263	14.98
M (Tensor interaction)	1.8298	19.08
AU (AV14 + UVII)	2.1433	9.78
FPS (UV14 + TNI)	1.8069	9.88

* See Table 2 of Cook *et al.*[12] for details.

It is well known that the masses and radii of rotating magnetized neutron stars are dependent on equations of state. The radius of a neutron star with a maximum mass is the smallest; a smaller mass, a slower rotation and/or a higher inner magnetic field would cause its radius to be larger.[12−14] Therefore the observational deduction of Eq.(12) should have some implications for proof-testing the equations of state available. Five equations of state have been focused[12] in the calculation for rapidly rotating neutron stars, the results of which are listed in Table 1 by interpolating between the tabulated points in Tables 9−23 of Cook *et al.*[12] for angular frequency $\Omega = 4 \times 10^3\ \mathrm{s}^{-1}$ (i.e., $P = 1.558$ ms). We see that the equations labelled "L" and "M" should be ruled out by the limits of Eq. (12). Certainly, we should also keep in mind that more equations of state may be killed by the inclusion of the inner magnetic field effect,[14] which is an interesting topic for future study.

Furthermore, two observational uncertainties may result in a derivation of stronger limits on the mass and radius of PSR 1937+21. The first is that observation would give a less steep position angle gradient (i.e. a smaller value of $(\mathrm{d}\psi/\mathrm{d}\phi)_{\max}$ than reality) due to smearing of finite sampling time, to the frequency dispersion in pulse arrival time,[15] and to the interstellar scattering[10] which may cause a stretching out of the longitude scale. In fact, $(\mathrm{d}\psi/\mathrm{d}\phi)_{\max} > 3$. A larger $(\mathrm{d}\psi/\mathrm{d}\phi)_{\max}$ (thus a smaller β according to Eq. (10)) favours a rough equality of the intensities of the main-pulse and inter-pulse. The second is that PSR 1937+21 might not be exactly an orthogonal rotator (i.e. α is not precisely 90°). It is possible that $\alpha < 90^\circ$. Both these uncertainties lead to a smaller value of $\rho\sqrt{P}$. We thus take $(\mathrm{d}\psi/\mathrm{d}\phi)_{\max}$ and α as two free parameters to calculate $\rho\sqrt{P}$. The calculated results are shown in Fig. 2. It is clear that stronger limits than those of Eq.(12) may rule out more neutron star equations of state, and probably conduce toward a strange star model for PSR 1937+21, based on Figs. 1 and 2.

In Fig. 2 we find $\rho\sqrt{P}$ is sensitively dependent on $(\mathrm{d}\psi/\mathrm{d}\phi)_{\max}$ if $(\mathrm{d}\psi/\mathrm{d}\phi)_{\max} \lesssim 10$; $\rho\sqrt{P} = 0.6$ if $(\mathrm{d}\psi/\mathrm{d}\phi)_{\max} = 4$ and $\alpha = 90^\circ$. In this case the limits of Eq. (12) should be modified as $M < 1.4\ M_\odot$ and $R < 6.6$ km. These limits inevitably lead to the conclusion that PSR 1937+21 is a strange star. Because of the observational uncertainties of $(\mathrm{d}\psi/\mathrm{d}\phi)_{\max}$ and α and of the importance of distinguishing a strange star in nature, we seriously table a proposal of detecting more accurate polarization signals from PSR 1937+21 to obtain an actual value, especially of $(\mathrm{d}\psi/\mathrm{d}\phi)_{\max}$. This could be possible by means of reducing the sampling time and increasing the total observation time.

If PSR 1937+21 is a strange star with small radius ($R < 10$ km), the old formulae[16] to calculate the surface magnetic field B and dipole magnetic moment μ_m should be modified as

$$\begin{aligned} B &= \left(\frac{c^3\rho P\dot{P}}{5\pi R}\right)^{1/2} \sim 9.3\times10^{19} R_5^{-1/2}(P\dot{P})^{1/2}\ \ \mathrm{G}, \\ \mu_\mathrm{m} &= \left(\frac{c^3\rho P\dot{P}R^5}{20\pi}\right)^{1/2} \\ &\sim 4.6\times10^{29} R_5^{5/2}(P\dot{P})^{1/2}\ \ \mathrm{G\cdot cm^3}, \end{aligned} \tag{13}$$

where R_5 is the stellar radius in 10^5 cm (1 km), since the density $\rho \sim 5\times10^{15}\ \mathrm{g\cdot cm^{-3}}$ has a very modest variation with radial distance of strange star,[17] $M \sim (4/3)\pi R^3\rho$, and the moment of inertia $I \sim (8/15)\pi\rho R_5$. We see from Eq. (13) that the calculated magnetic moment is strongly related to the radius although the observation-determined surface magnetic field is weakly related. For instance, if the radius of PSR 1937+21 is only 1 km, $B = 1.2\times10^9$ G, whilst $\mu_\mathrm{m} = 5.8\times10^{18}\ \mathrm{G\cdot cm^3}$ (typically, μ_m for millisecond pulsars is assumed to be $10^{26}\ \mathrm{G\cdot cm^3}$). This property favours the assumption that dipole magnetic structure dominates in the radio emission region.

Millisecond pulsars are currently believed to be of recycled-origin of normal pulsars which are spin down enough. However, motivated by the study of planet formation around PSR 1257+12, Miller and Hamilton[18] suggested that some and perhaps all isolated millisecond pulsars may have been born with high spin rates and low magnetic fields instead of having been recycled by accretion. This is understandable if we assume that part or all of the isolated millisecond pulsars are strange stars with smaller radii (thus smaller masses). (1) Turbulent convection in nuclear matter[19] or in strange quark matter[20] should be less prosperous in a proto-pulsar with lower mass than with higher mass in the Kelvin–Helmholtz cooling phase, the dynamo-created magnetic field is thus weaker. (2) Small radius favuors faster rotation since the centrifugal force becomes smaller while the gravitational force is larger (see Eq.(8)). A bimodal distribution of pulsar periods and magnetic fields may arise from various kinds of progenitor as well as complex mechanisms of supernova exploration. Therefore, it is conjectured that PSR 1937+21 and PSR 1257+12 (and possibly some or all of the isolated millisecond pulsars) may have weaker dynamo-originated magnetic field due to a less effective field magnification process.

References

[1] Huang Y F and Lu T 1997 *Chin. Phys. Lett.* **14** 314

[2] Chen C X and Zhang J L 2001 *Chin. Phys. Lett.* **18** 145

[3] Xu R X, Zhang B and Qiao G J 2001 *Astropart. Phys.* **15** 101

[4] Xu R X 2001 *Science & Technology Review* at press (in Chinese)

[5] Kapoor R C and Shukre C S 2000 preprint (astro-ph/0011386)

[6] Rankin J M 1990 *Astrophys. J.* **352** 247

[7] Alfvén H and Falthammer C G 1963 *Cosmical Electrodynamics* (Oxford: Clarendon)

[8] Kapoor R C and Shukre C S 1998 *Astrophys. J.* **501** 228

[9] Cordes J M and Stinebring D R 1983 *Astrophys. J.* **277** L53

[10] Gil J 1985 *Astron. Astrophys.* **143** 443

[11] Gil J 1983 *Astron. Astrophys.* **127** 267

[12] Cook G B, Shapiro S L and Teukolskey S A 1994 *Astrophys. J.* **424** 823

[13] Salgado M *et al* 1994 *Astron. Astrophys.* **291** 155

[14] Bocquet M *et al* 1995 *Astron. Astrophys.* **301** 757

[15] Liu X F and Wu X J 1999 *Acta Astrophys. Sin.* **19** 68 (in Chinese)

[16] Manchester R N and Taylor J H 1977 *Pulsars* (San Francisco, CA: Freeman)

[17] Alcock C, Farhi E and Olinto A 1986 *Astrophys. J.* **310** 261

[18] Miller M C and Hamilton D P 2001 *Astrophys. J.* (astro-ph/0012042) at press

[19] Thompson C and Duncan R C 1993 *Astrophys. J.* **408** 194

[20] Xu R X and Busse F H 2001 *Astron. Astrophys.* (astro-ph/0101011) at press