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What can the braking indices tell us about the nature of pulsars?

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Abstract

As a result of observational difficulties, braking indices of only six rotation-powered pulsars are obtained with some certainty. All these values are remarkably smaller than the value ($n = 3$) expected for pure magnetodipole radiation model. Various models have been proposed to explain this phenomenon such as introducing additional torques by particle outflow or accretion disk, changes in inclination angle or magnetic field strength, and field-reconnection process in pulsar magnetosphere. We re-examined the braking index based on previous research and suggest a constant gap potential drop in the open-field line region for pulsars with magnetospheric activity. New constraints on model parameters from observed braking indices are presented.

Keywords: Dense matter; Pulsars: general; Stars: neutron

1. Introduction

When describing the spin-down of pulsar, we usually make the assumption that the time derivative of the spin frequency $\dot{\Omega}$ is proportional to some power of spin frequency Ω ($=2\pi/P$),

$$\dot{\Omega} \propto \Omega^n, \quad (1)$$

where P is the spin period. The index n is usually assumed to be a constant which measures the efficiency of braking, i.e. the braking index. We can then define the braking index n and the second braking index (or jerk parameter) m as

$$n = \frac{\Omega\ddot{\Omega}}{\dot{\Omega}^2}, \quad m = \frac{\Omega^2\dddot{\Omega}}{\dot{\Omega}^3}. \quad (2)$$

By investigating n and m, we can obtain information on the pulsar's radiation and spin-down mechanisms. In the model that assumes spin-down is due to pure magnetodipole radiation, one has $n = 3$. If in turn, the main braking is caused by gravitational radiation, then one has $n = 5$. Any other contribution will provoke a modification of n from these "pure" values.

The fairly accurate timing property of pulsars gives us the opportunity to measure not only the spin frequency ν ($=1/P$) and the frequency derivative $\dot{\nu}$, but also the second derivative $\ddot{\nu}$ and even the third one. However, difficulties do exist in observation (e.g. Livingstone et al., 2007). As a result, braking indices of only six rotation-powered pulsars are obtained with some certainty, i.e. PSR J1846−0258 [$n = 2.65(1)$], PSR B0531+21 [2.51(1), the Crab pulsar], PSR B1509−58 [2.839(3)], PSR J1119−6127 [2.91(5)], PSR B0540−69 [2.140(9)] and PSR B0833−45 [1.4(2), the Vela pulsar], where the parentheses indicate the uncertainties of the last digits (Livingstone et al., 2006, and references therein, see Table 1 for more information of these six pulsars). All these six braking indices are smaller than the value ($n = 3$) predicted by pure dipole magnetic field configuration, which may suggest that other spin-down torques do work besides the energy loss via dipole radiation.

Pulsar's spin-down has been studied since its discovery, but why the braking indices are smaller than 3 still does not have a clear answer yet. In recent years, several new mechanisms are suggested to explain this discrepancy, e.g. the two-component model: spin-down due to both magnetodi-

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Table 1
Collected parameters of pulsars with measured braking indices

Name	P(s)	$\dot{P}$ (10^{-13} s/s)	n	B_0 (10^{13} G)	$L_{1.4\ \mathrm{GHz}}$ (mJy kpc^2)	L_x (10^{35} erg/s)	Ref.
B0531+21	0.033	4.23	2.51(1)	0.76	56.0	10[a]	1,2,8
B0540−69	0.050	4.79	2.140(9)	1.0	58.6	16[a]	1,3,8
B0833−45	0.089	1.25	1.4(2)	0.68	89.8	0.0050[a]	1,4,8
J1119−6127	0.408	40.2	2.91(5)	8.2	720	0.0055[b]	1,5,9
B1509−58	0.151	15.4	2.839(3)	3.1	18.2	0.20[a]	1,6,8
J1846−0258	0.324	71.0	2.65(1)	9.7	–[c]	4.1[b]	1,7,10

References: (1) the ATNF Pulsar Catalogue (http://www.atnf.csiro.au/research/pulsar/psrcat/), Manchester et al. (2005), (2) Lyne et al. (1993), (3) Livingstone et al. (2005a), (4) Lyne et al. (1996), (5) Camilo et al. (2000), (6) Livingstone et al. (2005b), (7) Livingstone et al. (2006), (8) Becker and Trümper (1997), (9) Gonzalez and Safi-Harb (2003), (10) Helfand et al. (2003).
[a] 0.1–4 keV.
[b] 0.5–10 keV.
[c] Radio quiet.

pole radiation and relativistic particle flow/wind (Dar, 1999; Allen and Horvath, 2000; Xu and Qiao, 2001; Wu et al., 2003; Contopoulos et al., 2006), the models with changing inclination angles (e.g. Blandford and Romani, 1988; Allen and Horvath, 1997a,b; Ruderman, 2005), the models with changing magnetic field strength (e.g. Blandford and Romani, 1988; Lin and Zhang, 2004; Lyne, 2004; Chen and Li, 2006), the models with additional torques due to accretion (e.g. Menou et al., 2001; Alpar et al., 2001; Chen and Li, 2006) and the model with field-reconnection in the magnetosphere (e.g. Contopoulos, 2006).

Recent observation of PSR B1931+24 by Kramer et al. (2006) suggests that magnetodipole radiation and particle flow both play important roles in pulsar's spin-down. In this paper, we focus on the two-component model proposed by Xu and Qiao (2001). Xu and Qiao (2001) assumed that the magnetic moment can be expressed as

$$\boldsymbol{\mu} = \boldsymbol{\mu}_{\perp} + \boldsymbol{\mu}_{\parallel}, \tag{3}$$

where

$$\mu = BR^3/2, \quad \mu_{\perp} = \mu \sin\alpha, \quad \mu_{\parallel} = \mu\cos\alpha \tag{4}$$

and α is the inclination angle. Considering different emission models such as inner vacuum gap with curvature radiation (VG, e.g. Ruderman and Sutherland, 1975 hereafter RS75), vacuum gap with resonant inverse Compton scattering (VG + ICS, e.g. Zhang et al., 2000), outer gap (OG, e.g. Cheng et al., 1986), and space charge limited flow (SCLF, e.g. Sturrock, 1971; Arons and Scharlemann, 1979), the braking index become a function of inclination angle (Xu and Qiao, 2001; Wu et al., 2003). The braking index in Xu and Qiao's (2001) model could range from a lower value (around 1–2, depending on different models) to 3. We also adopt this two-component assumption in this paper, but investigate in more detail the vacuum gap model. Assuming a constant potential drop ($\sim 10^{12}$ V) in the polar cap accelerating region, we have only one free parameter left. Comparing with observations, we present new constraints on model parameters. The particle density (but not the charge density) in the accelerating region could be $\sim 10^3$ to 10^4 times the (absolute value of) Goldreich–Julian charge density $|\rho_{GJ}|$ (Goldreich and Julian, 1969). Additionally, we show that a change of the moment of inertia I may also affect the braking index and make it smaller than 3.

2. A model with constant gap potential drop

In the two-component model (Xu and Qiao, 2001), the component $\mu_{\perp}$ is related to the magnetodipole radiation, while the other component $\mu_{\parallel}$, being related to the unipolar effect, could result in particle acceleration process modelled variously, such as VG, VG + ICS, OG, SCLF. The energy loss rate $\dot{E}$ of the two components have different dependencies on Ω. The magnetodipole component has $\dot{E}_{\perp} \propto \Omega^4$, which would alone induce a braking index of 3. The other component usually has a relatively weaker dependence on Ω (e.g. $\dot{E}_{\parallel} \propto \Omega^2$ for a constant gap potential drop due to the unipolar effect), which would induce by itself a braking index less than 3 ($n = 1$ in the unipolar case). Because of the different dependencies, the magnetodipole radiation is dominant when P is shorter while the other component becomes important when P is longer. The combination of these two may explain the observed braking index between 1 and 3. In summary, there are three variables/parameters: B, P and α. Here we firstly use the RS75 inner vacuum gap model for indication. The other models which can also be parameterized much in the same way will be discussed in Section 5.

The energy loss rates of dipole radiation and unipolar are

$$\dot{E}_{\mathrm{dip}} = -(2/3)c^{-3}\mu^2\Omega^4\sin^2\alpha \tag{5}$$

and

$$\dot{E}_{\mathrm{uni}} = -2\pi r_{\mathrm{pc}}^2\kappa|\rho_{\mathrm{GJ}}|\Phi = -c^{-1}\kappa BR^3\Omega^2\Phi\cos^2\alpha, \tag{6}$$

respectively, where $r_{\mathrm{pc}} \simeq (\Omega R^3/c)^{1/2}$ is the polar cap radius, $\rho_{\mathrm{GJ}} = B/(cP)$ is the Goldreich–Julian charge density (Goldreich and Julian, 1969), and κ is a coefficient related to particle density which will be constrained by observations. Since the plasma is mostly composed of $e^{\pm}$, we may write for κ,

$$\kappa \simeq \frac{n_+ + n_-}{|n_+ - n_-|}, \tag{7}$$

where n_+ and n_- are positron and electron number density, respectively (note that $\rho_{GJ} = |e|[n_+ - n_-]$, where e is the unit charge). We further assume that the parameter κ is a constant free parameter and relate it to observations below. Variable κ will be discussed in Section 5. Combining $\dot{E}_{dip}$ and $\dot{E}_{uni}$, we have

$$\dot{E} = I\Omega\dot{\Omega} = -\frac{2}{3c^3}\mu^2\Omega^4\eta, \tag{8}$$

where

$$\eta = \sin^2\alpha + 6c^2\kappa B^{-1}R^{-3}\Omega^{-2}\Phi\cos^2\alpha. \tag{9}$$

Define $f \equiv 6c^2\kappa B^{-1}R^{-3}\Phi$, we have

$$\eta = \sin^2\alpha + f\Omega^{-2}\cos^2\alpha. \tag{10}$$

The effective potential drop Φ of unipolar usually has a weak dependence on Ω (e.g. $\Phi \sim \Omega^{-1/7}$ in Xu and Qiao, 2001), or just a few $\times 10^{12}$ V (e.g. RS75; Usov and Melrose, 1995), i.e., just above the critical potential drop Φ_c. So we shall assume a constant potential drop $\Phi = 10^{12}$ V in the polar gap region. At the same time, a potential drop of 10^{12} V is also a widely accepted result from the pulsar death-line criterion (e.g. Zhang et al., 2000). This value is usually much less than the maximum potential drop that a pulsar can provide

$$\Phi_{max} = \frac{\Omega^2 BR^3}{2c^2}. \tag{11}$$

Comparison of Φ_{max} and Φ_c is presented in Fig. 1. The braking index and the second braking index then are

$$n = 3 - \frac{2}{f^{-1}\Omega^2\tan^2\alpha + 1} \tag{12}$$

and

$$m = 2n^2 - n + (n-3)(1-n). \tag{13}$$

The index n could be close to 1 for $f \gg \tan^2\alpha$ (e.g., small R and/or large κ).

Assuming a pulsar of $1.4M_\odot$ in mass and 10 km in radius with known ν, $\dot{\nu}$, $\ddot{\nu}$ and a constant potential drop $\Phi = 10^{12}$ V, we have three variables left: B, α and κ. Considering two constraints from Eqs. (8) and (12), we have only one free parameter left. Here we use κ as the free parameter and solve for B and α, i.e. B and α are obtained as functions of κ.

Our results are presented in Figs. 2–4 and are summarized as follows: (i) for a pulsar with a certain magnetic field and inclination angle, its braking index would evolve from 3 to 1 while its spin period P increase. (ii) The values of κ are most likely in the range 10^3–10^4 to produce reasonable values for B and α (Figs. 3 and 4). This is consistent with the hypothesis of Eq. (7).

In Fig. 3 we assume α is uniformity distributed between 0° and 90°. Observations (e.g. Lyne and Manchester, 1988) also suggest no clustering of inclination angles. We believe that the assumption of uniform distribution of inclination angles is not unreasonable within present status of the observations.

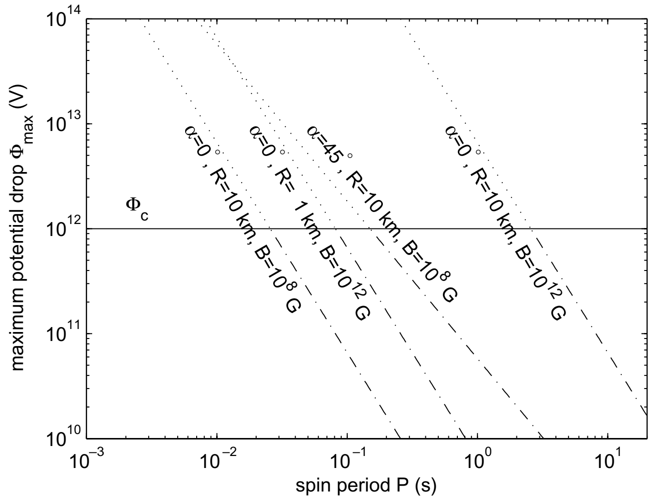

Fig. 1. The maximum potential drop Φ_{max} versus spin period P. Values of Φ_{max} are from Eq. (11) for $\alpha = 0$ and from Eq. (2) of Yue (2006) for $\alpha > 0$. The horizontal solid line denotes the critical potential drop Φ_c (we apply $\Phi_c = 10^{12}$ V in this figure). The dotted and dash-dotted lines denotes the maximum potential drop that can be achieved for a pulsar. Different inclination angle α, stellar radius R, and magnetic field strength B are denoted next to each line. If $\Phi_{max} < \Phi_c$ (plotted as dash-dotted line), the pulsar should be dead, i.e. no radio-activity. If $\Phi_{max} > \Phi_c$ (plotted as solid line), the pulsar should be active, i.e. being radio-loud. However, in this $\Phi_{max} > \Phi_c$ case, the effective potential drop Φ that accelerate particles could be around Φ_c and less than Φ_{max}. See Section 2 for details.

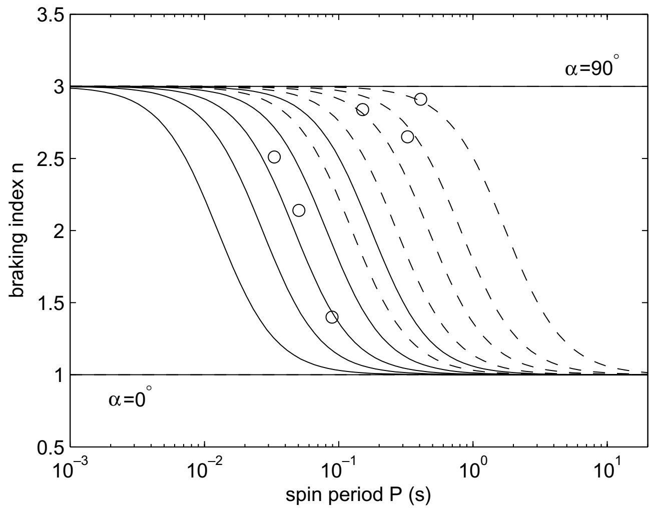

Fig. 2. Braking index n versus spin period P for the two-component model from Eq. (12). Here we apply magnetic dipole plus unipolar with constant potential drop $\Phi = 10^{12}$ V. $\kappa = 10^3$ are adopted. Solid line for magnetic field $B = 10^{12}$ G and dashed line for 10^{14} G. The six pulsars are plotted as circles. The lines from bottom to top correspond to α from 0° to 90° in 15° increments for each condition ($B = 10^{12}$ and 10^{14} G, respectively). Note that for $\alpha = 0°$ and 90°, solid and dashed lines are overlapped. A pulsar would evolve from left to right along a certain line while its braking index would drop from 3 to 1.

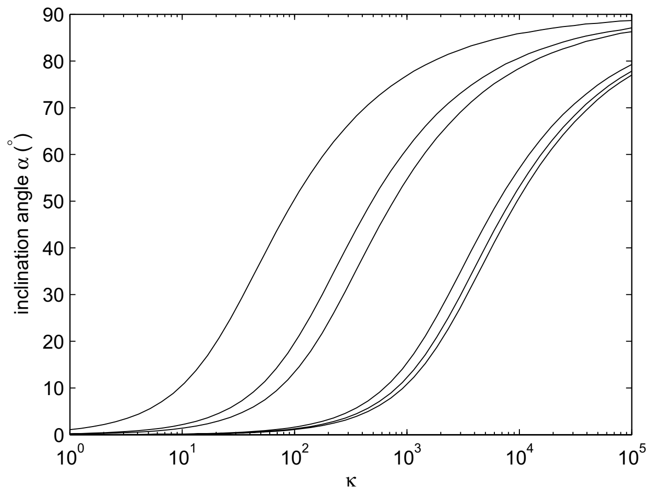

Fig. 3. Inclination angle α versus κ. Each line represents a pulsar. If we assume that α of the six pulsars are neither too big (close 90°) nor too small (close 0°), we have κ around 10^3, or say, in the range 10^2–10^4.

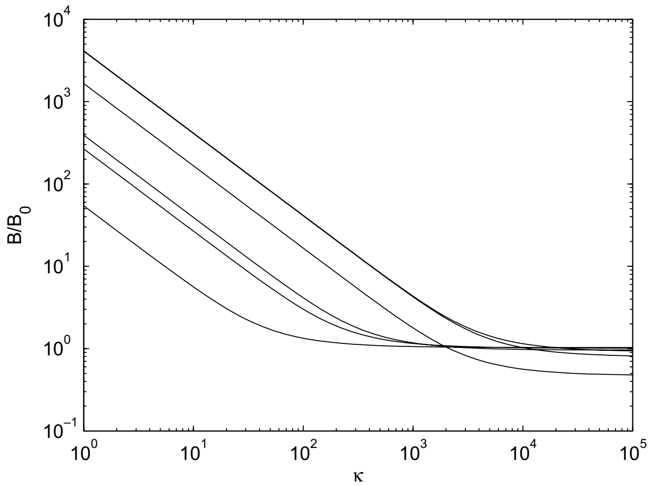

Fig. 4. The ratio of B/B_0 versus κ for the six pulsars with certain braking indices observed, where $B_0 = 6.4 \times 10^{19}(P\dot{P})^{1/2}$ (see Table 1 for detail values). Since B could not be too large, we should have $\kappa \gtrsim 10^3$. Together with Fig. 3, we have $10^3 \lesssim \kappa \lesssim 10^4$.

3. Further consideration of braking

In Section 2, we considered only emission but not pulsar interior. Inspired by the models which include changes of inclination angle and magnetic field strength, we consider pulsar interior properties in this section, especially on moment of inertia. The express of dipole radiation [Eq. (5)] can be rewritten as

$$\dot{\Omega} \propto B^2 R^4 \Omega^3 \sin^2 \alpha. \tag{14}$$

Any changes of B, α and/or I with Ω will induce a braking index $n \neq 3$. The changes of these three parameters are to some extent degenerated. $\dot{B}$ and $\dot{\alpha}$ have been considered in several models but the changes of moment of inertia (or stellar radius) are rarely mentioned. Including changes of I, energy loss rate is

$$\dot{E} = I\Omega\dot{\Omega} + \frac{1}{2}\dot{I}\Omega^2. \tag{15}$$

In Section 2, we only considered the first term $I\Omega\dot{\Omega}$ in Eq. (15) and omitted the second term $\dot{I}\Omega^2/2$, i.e. we set I as a constant. In this section we consider the effects of $\dot{I}$ in two cases: (i) volume conservative, i.e. the pulsar's volume is a constant and only the shape of the pulsar changes, (ii) volume non-conservative, i.e. assume that the pulsar's volume changes, or equivalently the stellar radius R changes. We show that the change of braking index is quite small in the first case, but not negligible in the second one.

3.1. The volume conservative case

A rotating star, or specifically a pulsar, can be approximately treated as an Maclaurin rotation ellipsoid when the spin period is not too small (Zhou et al., 2004). For pulsars, the criteria is $P \gg 1$ ms, so the rotation ellipsoid approximation are reasonable for all the six pulsar in the sample. In this case, the pulsar's volume is assumed to be a constant, i.e. the pulsar is assumed to be composed of incompressible fluid. The pulsar is spherical when $\Omega = 0$ and is a rotation ellipsoid when $\Omega > 0$. We have (Zhou et al., 2004)

$$I = I_0\left(1 + \frac{1}{3}e^2\right), \tag{16}$$

where

$$e = \frac{\Omega}{\sqrt{8\pi G\rho_*/15}}, \tag{17}$$

$\rho_*$is the average star density and I_0 is the star's moment of inertia when $\Omega = 0$. For Crab, the fastest one in the six, $e = 0.022$. Here we define

$$I = I_0(1 + k\Omega^2), \tag{18}$$

where $k = 5/(8\pi G\rho_*)$. Then we have

$$\dot{I} = \frac{\mathrm{d}I}{\mathrm{d}\Omega}\frac{\mathrm{d}\Omega}{\mathrm{d}t} = 2kI_0\Omega\dot{\Omega}. \tag{19}$$

Assuming $\dot{E}$ is a power-law function of Ω, we have

$$I\Omega\dot{\Omega} + \frac{1}{2}\dot{I}\Omega^2 = k_u\Omega^{u+1}, \tag{20}$$

where u is a constant (usually between 1 and 3) and k_u is a coefficient. Here u equals to braking index if we omit the $\dot{I}\Omega^2/2$ term. With Eqs. (18)–(20), we have

$$\dot{\Omega} = \frac{k_u}{I_0}\frac{\Omega^u}{1 + 2k\Omega^2} \tag{21}$$

and

$$\ddot{\Omega} = \frac{k_u}{I_0}\frac{\Omega^{u-1}\dot{\Omega}}{1 + 2k\Omega^2}\left(u - \frac{4k\Omega^2}{1 + 2k\Omega^2}\right). \tag{22}$$

Then we have for the braking index

$$n' = \frac{\ddot{\Omega}\Omega}{\dot{\Omega}^2} = u - \frac{4k\Omega^2}{1+2k\Omega^2}. \tag{23}$$

Or effectively we have the difference between the braking index when assuming a constant I and the braking index when considering $\dot{I}$,

$$\Delta n = n - n' = u - n' = \frac{4k\Omega^2}{1+2k\Omega^2}. \tag{24}$$

For Crab, $k\Omega^2 = e^2/3 = 1.6\times10^{-4} \ll 1$, $\Delta n = 6.5\times10^{-4}$, which is negligible. Since Crab is the fastest among the six pulsars, this effect can be omitted for all the six pulsars.

3.2. The volume non-conservative case

In the above subsection we show that the braking index is almost a constant in the volume conservative case, because the change of I is very small. Here we consider a more effective process which leading to larger $\dot{I}$, i.e. the star radius R changes. Here we assume that I is a function of Ω. Applying the first order approximation, we can express dI/dΩ in a dimensionless way,

$$\frac{dI}{d\Omega} = f_1\frac{I}{\Omega}, \tag{25}$$

where I is the current moment of inertia and f_1 is a dimensionless coefficient. Together with Eq. (20), we have

$$\dot{\Omega} = \frac{k_u}{I}\frac{\Omega^u}{1+f_1/2} \tag{26}$$

and then

$$\ddot{\Omega} = \frac{k_u}{I}\frac{\Omega^{u-1}\dot{\Omega}}{1+f_1/2}(u-f_1). \tag{27}$$

The braking index is then

$$n = u - f_1. \tag{28}$$

The numerical value of f_1 should be of the order of 1 to make braking index significantly smaller than u (u is the braking index if $\dot{I}=0$ is assumed), i.e. [from Eq. (25)]

$$\frac{dI}{d\Omega}\left(=\frac{\dot{I}}{\dot{\Omega}}\right) \sim \frac{I}{\Omega}. \tag{29}$$

For the Crab pulsar we have,

$$\dot{I} \sim I\frac{\dot{\Omega}}{\Omega} = 4\times10^{-4} I \text{ year}^{-1}. \tag{30}$$

This means the Crab pulsar's moment of inertia should have changed by ~1% in the past 30 years. It seems large but is not totally impossible. Here we propose glitch/star quake as one possible mechanism. The post glitch behavior of pulsars (e.g. Lyne et al., 1993) shows increasing of spin-down rate after glitches. This phenomena, which may be caused by increasing of spin-down torque (Link et al., 1992, 1998), can also be understood by decreasing of I. A pulsar's radius would decrease as it spins down because the centrifugal force decreases. However, a pulsar, especially a quark star, may become solid (or partly solid) shortly after its birth. Elastic strain energy would then develop as a solid star spins down and acts against the decrease of the star's radius. Star quakes would reconfigure the distribution of matter and cause the decrease of stellar volume (e.g. Xu et al., 2006). Vela is most likely this case. It is worth noting that the "observed" field increase of the Crab pulsar (e.g. Lyne, 2004) could also arise from the shrinking of pulsars after quakes.

4. To test the model

The conjecture presented in Xu and Qiao (2001), that pulsars are torqued by both magnetodipole radiation and particle ejection, is actually phenomenological, with an assumption that the spin-down power equals to a sum of energy losses due to $\mu_\parallel$ and $\mu_\perp$ ($\boldsymbol{\mu} = \boldsymbol{\mu}_\perp + \boldsymbol{\mu}_\parallel$ is the total magnetic momentum). It is worth noting that this idea could be tested by future observations. We propose several possible ways to test the model for pulsar braking, with some predictions for braking indices of different kinds of pulsars.

For dead pulsars (i.e. pulsars without magnetospheric activity), only magnetodipole torque may exist. We would expect braking index values of 3 for them in our model. For active pulsars, both magnetodipole and particle outflow torque may exist. We would expect braking index values less than 3. This point, as an prediction of our model, could be tested by possible future observation of several kinds of pulsar-like stars in different bands, especially in X-ray bands for radio quiet pulsars. Pulsars could be observed in both radio and X-ray bands when they are radio-on, and maybe in X-ray band only when they are radio-off. Thus the two-component conjecture could be checked by future radio and X-ray timing observations. Here we propose several candidates that could be observed in order to test our model, i.e. part time pulsar B1931+24, dim thermal neutron stars, and rotating radio transients.

The part time pulsar, PSR B1931+24, has different slowdown rate when it is radio-on and -off: $\dot{\nu}_{\rm on} = 16.3(4)\times10^{-15}$ Hz s^{-1} and $\dot{\nu}_{\rm off} = 10.8(2)\times10^{-15}$ Hz s^{-1} (Kramer et al., 2006). We should expect a braking index of $n=3$ when it is radio-off if no particle outflow exists during this phase and $n<3$ when it is radio-on since particle outflow torque acts on the pulsar. From Eq. (8), we have

$$\frac{\dot{\Omega}_{\rm on}}{\dot{\Omega}_{\rm off}} = \frac{\dot{\nu}_{\rm on}}{\dot{\nu}_{\rm off}} = 1 + f\Omega^{-2}\tan^{-2}\alpha. \tag{31}$$

Together with Eq. (12), we can get $n_{\rm on} = 2.33$ and $n_{\rm off} = 3$ for PSR B1931+24. The value 2.33 is quite distinguishable from 3. If weak particle wind exists when it is radio-off, the braking index $n_{\rm off}$ could be a little bit smaller than 3. We expect this difference of indices could be checked by future long period observations of PSR B1931+24.

For dim thermal neutron stars (Haberl, 2005), which are suggested to be radio quiet, no magnetospheric activity should exists. We should expect braking index values of 3. The braking indices of dim thermal neutron stars, though hard to measure, may be get from future X-ray observations.

Rotating radio transients (RRATs, McLaughlin et al., 2006) are radio quiet for most of the time and only show rare radio bursts – one pulse in several hundreds to several thousands spin periods. Because the particle outflow only exist when they are on (bursting), particle wind braking exist for much shorter time than magnetodipole braking. So magnetodipole braking dominates for RRATs. Therefore the braking indices of RRATs should be quite close to 3.

5. Conclusion and discussion

We present a calculation of pulsar's braking index considering both magnetodipole radiation and RS75 inner vacuum gap with a constant potential drop. Assuming a constant potential drop $\Phi = 10^{12}$ V, we obtain that the particle density in the polar cap region should be $\sim 10^3$ to 10^4 times $|\rho_{GJ}|$, i.e. $\kappa = (n_+ + n_-)/|n_+ - n_-| \sim 10^3$–$10^4$. The dynamical implication of this result in pulsar electrodynamics is worth researching in the future. We also show that the effects of $\dot{I}$ on the braking index could not be omitted if $\dot{I}$ is of the order of $10^{-4} I$ year^{-1}.

In our results, we need $\kappa \gg 1$. This means that the $e^\pm$ pair plasma could be accelerated in gaps with potential drop Φ. Though RS75-type vacuum gap may result in sparking, which would be necessary for explaining drifting subpulses, it is still possible that pair plasma could be accelerated (i) above the vacuum gap and/or (ii) in annular gaps (Qiao et al., 2004) or slot gaps (e.g. Harding and Muslimov, 1998). Although the effective potential drop could be a constant (about 10^{12} V, which may not depend on spin period), the accelerating region could be quite long: e.g. from the stellar surface to almost the light cylinder in the annular gap model (Qiao et al., in preparation). In such "long gaps", secondary $e^\pm$ pairs produced might be efficient so as to induce a high particle density, e.g. $\sim 10^3$ $|\rho_{GJ}|$, and results in a high electric current in our model. If not only VG but also ICS and/or OG exists (e.g. VG + ICS model in Zhang et al., 2000), i.e. the effective potential drop is larger than the potential drop of the inner gap, and the particle density could be considerably smaller. In the vacuum gap model, once the gap potential drop Φ reaches critical value Φ_c (=a few $\times 10^{12}$ V), sparks form and produce $e^\pm$ plasma and hence current flow. This process stops the increasing of the gap potential drop. Thus the effective accelerating potential drop could be just around Φ_c. However, we would like to note that Φ might changes slightly with Ω (e.g. $\Phi \sim \Omega^{-1/7}$ in Xu and Qiao, 2001). This effect would change the braking index by a factor, e.g. $\Delta n \sim -1/7$ if $\Phi \sim \Omega^{-1/7}$. Other models such as VG, VG + ICS, OG, and SCLF could also parameterized the same way if their spin-down power could be expressed as functions of P, B and α. However the minimum value of the braking index might be less than 1.

The factor κ is assumed to be a constant in this paper. We would like to note that κ might also be a function of Ω and then the variation range of braking index n would changes. Among the six pulsars, only the Vela pulsar has $n < 2$, which is not obtained from phase-connected solution; the rest five n derived from phase-connected solutions are all larger than 2 (Livingstone et al., 2007). If Vela pulsar's n from extrapolation (Lyne et al., 1996) does not reflect the internal braking index (e.g. affected by decreasing of I caused by glitch), this would open up room for a dependence of κ with Ω, e.g. κ might be proportional to Ω and pulsar braking index could be in the range 2–3.

The second braking indices of two pulsars have been measured, i.e. Crab ($m = 10.23 \pm 0.03$, Lyne et al., 1993) and PSR B1509−58 ($m = 18.3 \pm 2.9$, Livingstone et al., 2005b). The theoretical values from Eq. (13) are $m = 10.9$ for Crab and $m = 13.6$ for PSR B1509–58, which are not compatible with the observations within the error bar. However, this discrepancy might not be a serious problem, because the braking indices and the second braking indices are affected by several processes such as timing noise, very small glitches and changes of surrounding environment (e.g. interaction with ISM), which are not clearly understood. The second braking index m is more complex than the first braking index and should be investigated when more m values are available.

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