



Quakes in solid quark stars

A.Z. Zhou^a, R.X. Xu^{b,*}, X.J. Wu^b, N. Wang^c

^a Shanghai Astronomical Observatory, Chinese Academy of Sciences, Shanghai 200030, China

^b School of Physics, Peking University, Beijing 100871, China

^c Urumqi Station, The National Astronomical Observatories, The Chinese Academy of Sciences, Urumqi 830011, China

Received 3 May 2004; received in revised form 22 May 2004; accepted 26 May 2004

Available online 19 June 2004

Abstract

A starquake mechanism for pulsar glitches is developed in the solid quark star model. It is found that the general glitch natures (i.e., the glitch amplitudes and the time intervals) could be reproduced if solid quark matter, with high baryon density but low temperature, has properties of shear modulus $\mu = 10^{30-34}$ erg/cm³ and critical stress $\sigma_c = 10^{18-24}$ erg/cm³. The post-glitch behavior may represent a kind of damped oscillations.

© 2004 Elsevier B.V. All rights reserved.

PACS: 97.60.Gb; 97.60.Jd; 11.80.Fv

Keywords: Pulsars; Neutron stars; Elementary particles

1. Introduction

Pulsars are unique objects, with which all types of elemental interaction could be tested extremely. However, the most elementary question relevant is still open: *What is the nature of pulsars?* It is conventionally thought that pulsars are simply a kind of boring big “nuclei”—neutron stars, but more and more attention is paid to the quark star model for pulsars [1,2] since *no* convincing work, neither in theory from first principles nor in observation, has confirmed Baade–Zwicky’s original idea that supernovae produce neutron stars. The bare quark

surface is suggested to be a new probe for identifying quark stars with strangeness, and possible observational evidence for bare strange stars appears: the drifting subpulses of radio pulsars, ultra-high luminosity of soft γ -ray repeaters, non-atomic thermal spectra of isolated “neutron” stars [3]. However, can the bare strange star model reproduce most of the general features of pulsars (especially glitches)?

The observation of free precession in PSR B1828-11 [4] and PSR B1642-03 [5] challenges astrophysicists today to re-consider the internal structure of radio pulsars [6]. The current model for glitches involves neutron superfluid vortex pinning and the consequent fluid dynamics. However, the pinning should be much weaker than predicted in the glitch models, otherwise the vortex pinning will damp out the precession on timescales being much

* Corresponding author. Tel.: +86-106-275-8631; fax: +86-106-276-5031.

E-mail address: rxxu@bac.pku.edu.cn (R.X. Xu).

smaller than observed. In addition the picture that a neutron star core contains coexisting neutron vortices and proton flux tubes is also inconsistent with observations of freely precessing pulsars [7]. It is then supposed that the hydrodynamic forces presented in a precessing star are probably sufficient to unpin all of the vortices of the inner crust [8] since a definitive conclusion on the nature of vertex pinning has not been reached yet due to various uncertainties in the microscopic physics. But recently, Levin and D’Angelo [9] studied the magnetohydrodynamic (MHD) coupling between the crust and the core of a rotating neutron star, and found that the precession of PSR B1828-11 should decay over a human lifetime. This well-defined MHD dissipation should certainly be important in order to test the stellar models.

An alternative way to understand both glitch and free-precession could be through the suggestion that radio pulsars are solid quark stars [10,11]. A solid quark star is just a rigid-like body, no damping occurs, and the solid pulsar model may survive future observational tests if the free precession keeps the same over several tens of years. A neutron star could not be in a solid state, whereas a cold quark star could be. Such a solid state of quark matter could be very probably Skyrme-like¹ [13,14], the study of which may help us to understand dense quark matter with low temperature.

Fluid strange-star (even with possible crusts) models were noted to be inconsistent with the observations of pulsar glitches more than one decade ago [15]. Modifications with the inclusion of possible stable particles to form a differentiated structure of so-called strange pulsars was also suggested [16], but is not popular because of a disbelief in the employed physics [17,6]. However, can a fully solidified quark star proposed [10] really reproduce the glitch behaviors observed? One negative issue is that giant glitches are generally not able to occur at an observed rate in a solid neutron star [18]. Nonetheless, more strain energy could be stored in a solid quark star due to

an almost uniform distribution of density (the density near the surface of a bare strange star is $\sim 4 \times 10^{14}$ g/cm³) and high shear modulus introduced phenomenologically for solid quark matter with strangeness. More energy is then released, and this may enable a solid pulsar to glitch frequently with large amplitudes. Furthermore, the post-glitch behaviors may represent damped vibrations. In this paper, we try to model glitches in a starquake scenario of solid quark stars.

2. The model

A quake model for a star to be mostly solid was generally discussed by Baym and Pines [18] and others [19,20], who parameterized the dynamics for solid crusts, and possible solid cores, of neutron stars. Strain energy develops when a solid star spins down until a quake occurs when stellar stresses reach a critical value. Two possibilities were considered previously. Ruderman [21] assumed that the entire strain energy is relieved in the quake, while Baym and Pines [18] suggested only part of the stress is released and that the plastic flow is negligible. We present a third possibility that, during a quake, the entire stress is almost relieved at first when the quake cracks the star in pieces of small size (the total released energy E_t may be converted into thermal energy E_{therm} and kinematic energy E_k of plastic flow, $E_t = E_{\text{therm}} + E_k$), but the part of E_k might be re-stored by stress due to the anelastic flow (i.e., the kinetic energy is converted to strain energy again). As shown in Fig. 1, a quark star may solidify with an initial oblateness ε_0 ; stress (to be relative to ε_0) increases as the star loses its rotation energy, until the star reaches an oblateness ε_{+1} . A quake occurs then, and the reference point of strain energy should be changed to ε_1 (the oblateness of a star without shear energy) at this moment. Damped vibrations in the potential field (see the inserted up-right part of Fig. 1) occur therefore. After a glitch, the star solidifies and becomes elastic body again. No memory of the first glitch affects the second one.

The density of quark stars with mass $< \sim 1.5M_\odot$ can be well approximated to be uniform [22]. As a

¹ Skyrme [12] considered baryons as solitons. The n -quark clusters might also be described as solitons in a similar way.

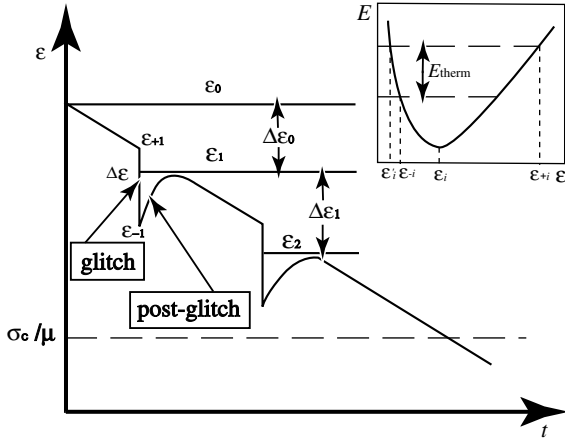


Fig. 1. The oblateness as a function of time, $\varepsilon(t)$. The horizontal solid lines are for the reference points of the strain. The steps, drawn very exaggeratedly, are starquakes. No starquake occurs anymore when ε falls below σ_c/μ (i.e., the stress can not be large enough for a quake).

star, with an initial value ε_0 , slows down, the expected ε decreases with increasing period. However, the rigidity of the solid star causes it to remain more oblate than it would be had it no resistance to shear. The strain energy in the star reads [18]

$$E_{\text{strain}} = B(\varepsilon - \varepsilon_0)^2, \quad (1)$$

and the mean stress σ in the star is

$$\sigma = \left| \frac{1}{V} \frac{\partial E_{\text{strain}}}{\partial \varepsilon} \right| = \mu(\varepsilon_0 - \varepsilon), \quad (2)$$

where ε is stellar oblateness,² $V = 4\pi R^3/3$ is the volume of the star, and $\mu = 2B/V$ is the mean shear modulus of the star. We note that, in the following calculations, the stress-increase developed by the decrease of oblateness (due to the spin down) is only included. However, azimuthal stress due to the general relativistic effect [11] (maybe similar to the frame-dragging effect in vacuum) of rotating solid stars may contribute significantly,

² The eccentricity e is defined by $e^2 = 1 - c^2/a^2$ (a and c are the semimajor and semiminor axes, respectively). The oblateness (or ellipticity) $\varepsilon \equiv (I - I_0)/I_0$ (I_0 is the non-rotating, spherical moment of inertia) is related to e through $\varepsilon = (1 - e^2)^{-1/3} - 1 \approx e^2/3$.

though, unfortunately, the theoretical answers to elastic relativistic-stars with rotation are very difficult to be worked out.

The total energy of such a rotating star with mass M and radius R is the sum of the gravitational energy E_{gravi} , the rotation energy E_{rot} , the strain energy E_{strain} , the bulk energy E_v , and the surface energy E_s ,

$$\begin{aligned} E &= E_{\text{gravi}} + E_{\text{rot}} + E_{\text{strain}} + E_v + E_s \\ &= E_0 + A\varepsilon^2 + L^2/(2I) + B(\varepsilon - \varepsilon_i)^2 + E_v + E_s, \end{aligned} \quad (3)$$

where ε_i is the reference oblateness before the $(i + 1)$ -th glitch occurs, $E_0 = -3M^2G/(5R)$, I is the moment of inertia, $L = I\Omega$ is the stellar angular momentum, $\Omega = 2\pi/P$ (P the rotation period), and the coefficients A and B measure the gravitational and strain energies [18], respectively,

$$A = \frac{3}{25} \frac{GM^2}{R}, \quad (4)$$

$$B = \frac{2}{3} \pi R^3 \mu. \quad (5)$$

The changes of E_v and E_s are much smaller than that of E_{gravi} , E_{rot} , or E_{strain} when a star spins down, according to a mass formula for strange quark matter to be analogous to the Bethe–Weizsacher semi-empirical mass function [23]. We therefore neglect E_v and E_s in the following calculations.

By minimizing the total energy E , a real state satisfies (note that $\partial I(\varepsilon)/\partial \varepsilon = I_0$),

$$\varepsilon = \frac{I_0 \Omega^2}{4(A + B)} + \frac{B}{A + B} \varepsilon_i. \quad (6)$$

The reference oblateness is assumed, by setting $B = 0$ in Eq. (6), to be

$$\varepsilon_i = I_0 \Omega^2 / (4A). \quad (7)$$

A star with oblateness of Eq. (7) is actually a Maclaurin sphere. When the star spins down to Ω , the stress develops to

$$\sigma = \mu \left[\frac{A}{A + B} \varepsilon_i - \frac{I_0 \Omega^2}{4(A + B)} \right], \quad (8)$$

according to Eq. (2). A glitch takes place if the stress is greater than a critical one, $\sigma > \sigma_c$.

The first quake is characterized by a sudden shift of ε , the amount of which is

$$\Delta\varepsilon = \varepsilon_{+1} - \varepsilon_{-1}, \quad (9)$$

where the reference point is changed from ε_0 to ε_1 . Due to the conservation of stellar angular momentum, the sudden change in the oblateness results in an increase of spin frequency,

$$\frac{\Delta\Omega}{\Omega} = -\frac{\Delta I}{I} = \Delta\varepsilon. \quad (10)$$

Starquakes would also produce a thermal energy dissipation during glitches that could be expected to be observable as an increase of X-ray luminosity soon after glitch. According to the conservation of energy, one obtains from Eq. (3),

$$\begin{aligned} & A\varepsilon_{+1}^2 + 0.5\varepsilon_{+1}I_0\Omega_1^2 + B(\varepsilon_{+1} - \varepsilon_0)^2 \\ &= A\varepsilon_{-1}^2 + 0.5\varepsilon_{-1}I_0\Omega_1^2 + B(\varepsilon_{-1} - \varepsilon_1)^2 + E_{\text{thermal}}, \end{aligned} \quad (11)$$

where Ω_1 , which can be obtained by $\sigma = \sigma_c$ from Eq. (8), is the spin frequency when the first quake occurs, E_{thermal} is the released energy in a starquake. The observed glitch size $\Delta\Omega/\Omega$ can be calculated from Eqs. 6 to 11. The calculations, based on these equations, are parameterized by five input quantities: M , ρ , μ , σ_c and E_{thermal} , and two of them are fixed to be $M = 1.4M_\odot$ and $\rho = 4 \times 10^{14} \text{ g/cm}^3$.

Fig. 2 shows the results for various μ and σ_c . We see that, for $\sigma_c > \sim 10^{22} \text{ erg/cm}^3$, $\Delta\Omega/\Omega$ decays to be a constant for pulsars with large rotation periods. Small μ may generally result also in a small jump of $\Delta\Omega/\Omega$ for a certain period P . However, for $\sigma_c < \sim 10^{22} \text{ erg/cm}^3$, the asymptotic line is reached, which means that $\ln(\Delta\Omega/\Omega) \propto -\ln P$. We choose $E_{\text{therm}} = 10^{36} \text{ erg/cm}^3$ in computation, but the curves do not depend sensitively on E_{therm} . It is found that the glitch amplitude, $\Delta\Omega/\Omega$, could be as high as observed if the shear modulus $\mu \sim 10^{30-34} \text{ erg/cm}^3$. Though the quark matter with low temperature and high baryon density is focused recent years, it is still impossible to determine the properties of such QCD phase by first principles (most of the calculations are started from QCD phenomenological models). One of the proposed states of strange stars at low temperature could be a solid state [10] (and also the 2nd par-

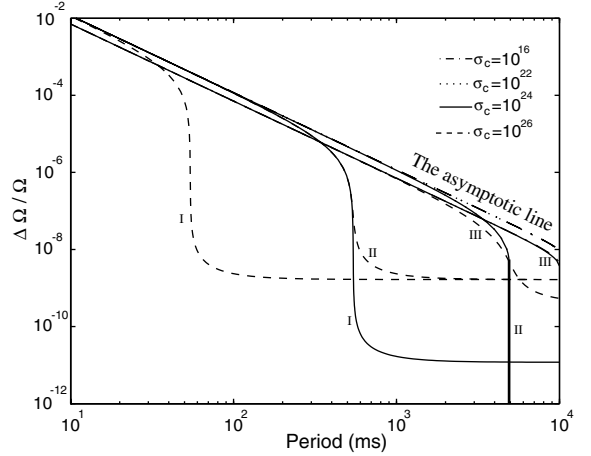


Fig. 2. The fractional increase in the spin frequency during a glitch, $\Delta\Omega/\Omega$, vs. the rotation period, P . The lines labeled I, II, and III are for different shear modulus, μ : I: $\mu = 10^{30} \text{ erg/cm}^3$, II: $\mu = 10^{32} \text{ erg/cm}^3$, and III: $\mu = 10^{34} \text{ erg/cm}^3$. The dash-dot, dotted, solid and dashed lines are for $\sigma_c = 10^{16}$, $\sigma_c = 10^{22}$, $\sigma_c = 10^{24}$ and $\sigma_c = 10^{26} \text{ erg/cm}^3$, respectively. We observe that the curves tend towards the asymptotic line when $\sigma_c < \sim 10^{22} \text{ erg/cm}^3$. The stellar mass and mean density are chosen to be mass $M = 1.4M_\odot$ and density $\rho = 4 \times 10^{14} \text{ g/cm}^3$ in the calculations.

agraph of Section 4), but the calculations on solid state is much more difficult than that of fluid one. Nevertheless, we may estimate the shear modulus μ originated only by electric interaction between charged n -quark clusters (n : the quark number in a cluster) and a uniform background of electrons, through a similar quantum mechanical calculation of metals [24]. The effective shear modulus, averaged over polarizations and directions, can be well fitted by $\mu \sim 0.12N(Ze)^2/a$ in asymptotic case, where Z is the charge of quark-clusters, N the cluster number density and a the separation between clusters [25]. For strange quark matter with baryon number density n_B and electron number density to be $\sim 10^{-3}$ that of quarks, one comes to

$$\mu \simeq 10^{28} \left(\frac{n_B}{2n_0} \right)^{4/3} \left(\frac{n}{10^3} \right)^{2/3} \text{ erg/cm}^3, \quad (12)$$

where $n_0 = 0.16$ baryons per fm^3 is the nuclear saturation density. This result can be regarded as a low limit modulus of solid strange quark matter since the van der Waals type color interaction,

with a high coupling constant (to be corresponding to the electric charge e in electromagnetic interaction), may result in a larger shear modulus. We might then conclude that μ in the range of 10^{30-34} erg/cm³ is not impossible. If the kilohertz quasi-periodic oscillations are relevant to the global oscillation behavior of solid quark stars, the shear modulus could be [10] $\mu \sim 10^{32}$ erg/cm³, which is much larger than the value ($\sim 10^{28}$ erg/cm³) of neutron star crust. This could be unsurprise due to much high density.

After the first quake, from Eq. (8), the stress in the star builds up again, at a rate of

$$\dot{\sigma} = -\mu\dot{\varepsilon} = -\frac{\mu U_0}{2(A+B)}\Omega\dot{\Omega}, \quad (13)$$

which is almost a constant for a certain pulsar with P and \dot{P} during a period when the effect of \dot{P} is negligible. Another quake occurs after a time of

$$t_q = \sigma_c / \dot{\sigma}. \quad (14)$$

The calculation for certain P and \dot{P} is presented in Fig. 3. For a certain set of $\{P, \dot{P}\}$, the interval t_q become longer for a higher value of σ_c . It is, however, not necessary to expect that a radio

pulsar should jump its rotation periodically, because of no reason to show that the critical value σ_c (and maybe μ) keeps a constant during its life. The time separation t_q may present a quasi-periodic nature if σ_c has a Gaussian distribution.

3. The post-starquake behaviors

When i -th quake occurs in a pulsar, the oblateness decreases to ε_{-i} at first, while the reference point is changed to ε_i from ε_{i-1} . Soon after the rotational jump ($\varepsilon_{+i} \rightarrow \varepsilon_{-i}$), the star has a tendency to reach the equilibrium oblateness ε_i . Certainly, this recovery process depends on the difference, $\varepsilon_{-i} - \varepsilon_i$, which is shown in Fig. 4 as a function of E_{therm} . Increasing oblateness can obviously contribute an additional spin down role besides that of magnetodipole radiation, which results in larger \dot{P} . Such a post-starquake could be observed as a recover behavior during post-glitch. From Fig. 4, one can see the values of $\varepsilon_{-i} - \varepsilon_i$ are generally not sensitive to both E_{therm} and σ_c .

The recovery of $\varepsilon_{-i} \rightarrow \varepsilon_i$ is actually a complex process, in which both elastic revert and plastic flow could not be negligible. Nevertheless, this

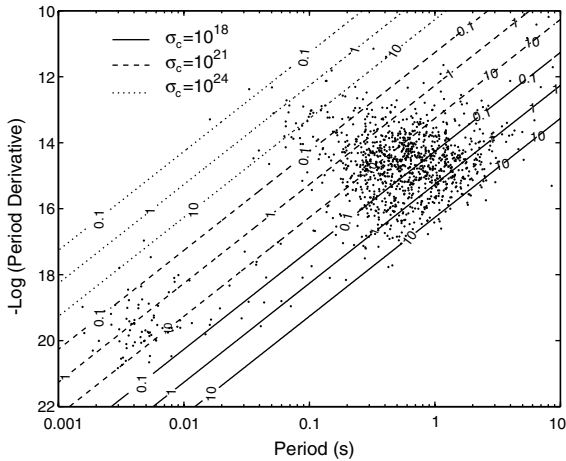


Fig. 3. Contour lines of the time separation between two quakes, t_q , on the P – \dot{P} diagram. This glitching interval time t_q is labelled to the lines in unit of years. The parameters are taken as: $M = 1.4M_\odot$, $\rho = 4 \times 10^{14}$ g/cm³, and $\mu = 10^{32}$ erg/cm³. Results with three values of σ_c (10^{18} , 10^{21} , and 10^{24} erg/cm³) are for indications. The pulsar data were downloaded from <http://www.atnf.csiro.au/research/pulsar/psrcat>.

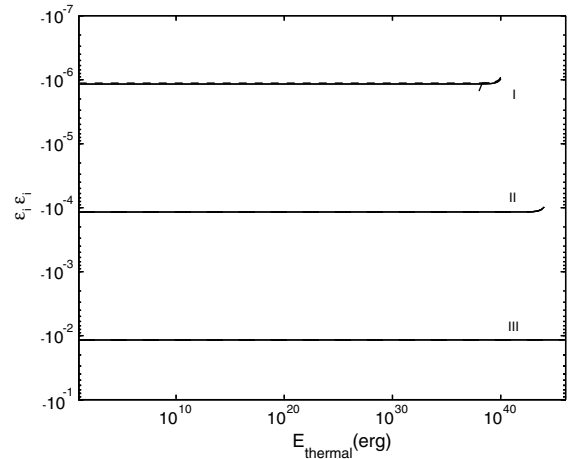


Fig. 4. The difference of $\varepsilon_{-i} - \varepsilon_i$ as a function of E_{therm} . The lines are grouped for different periods; “I”: $P = 1$ s, “II”: $P = 0.1$ s, and “III”: $P = 0.01$ s. The solid and dashed lines are for $\sigma_c = 10^{16}$ and $\sigma_c = 10^{24}$ erg/cm³, respectively, which are almost the same for certain period P . Other parameters chosen are: $M = 1.4M_\odot$, $\rho = 4 \times 10^{14}$ g/cm³, and $\mu = 10^{32}$ erg/cm³.

recovery process might be an analog of damped oscillations, with a mathematical description of

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0, \quad (15)$$

with $x \equiv \varepsilon - \varepsilon_i$. The second term in Eq. (15) arise from the anelastic effect during the recovery. The solution of Eq. (15) depends on the relative strength of the elastic spring and that of the damping. It is well known that three possible solutions exist; the so-called underdamped case: $\beta < \omega_0$, the critical case: $\beta = \omega_0$, and the overdamped case: $\beta > \omega_0$; where ω_0 is the intrinsic frequency. One comes then to $\omega_0 \sim \sqrt{\mu R/M}$ by dimensional analysis (force $\sim \sigma R^2$ and displacement $\sim xR$).

In all the three cases, the oscillation damps by a factor of $\sim \exp(-\beta t)$, with a typical time of $\tau = 1/\beta$. We are just to discuss the critical damped motion below for an indication. In this case, $\omega_0 = \beta$, the time evolution of ε after the occurrence of starquake can be solved by Eq. (15),

$$\varepsilon(t) = \varepsilon_i + (\varepsilon_{-i} - \varepsilon_i)(1 + t/\tau)e^{-t/\tau}. \quad (16)$$

4. Conclusions and discussion

A starquake model for pulsar glitches is developed in the regime of solid quark stars, and it is found that the general glitch behaviors (i.e., the glitch amplitude $\Delta\Omega/\Omega$ and the time interval t_q) could be reproduced if solid quark matter has properties of shear modulus $\mu = 10^{30-34}$ erg/cm³ and critical stress $\sigma_c = 10^{18-24}$ erg/cm³. It is suggested that the post-glitch process could be described as damped oscillations, especially in the critical and the overdamped cases.

Anyway, this is only a primary and simplified study of quakes in solid quark stars, more elaborate work, with possible modifications, on both quake and post-quake processes is necessary in order to understand the nature of solid quark matter through glitching pulsars.

We are dealing with solid quark stars in this paper. The quark Cooper pairing of the BCS type is suggested in quark matter of low-temperature but high baryon density, which may result in a color superconducting state [26], with a large pairing gap

on the order of 100 MeV. This kind of condensation in momentum space takes place in case of same Fermi momenta; whereas ‘‘LOFF’’-like state may occur if the Fermi momenta of two (or more) species of quarks are different [27]. For three flavors of massless quarks, all 9 quarks pair in a pattern which locks color and flavor symmetries, as called color–flavor locking (CFL) state [28]. However, for such quark matter, there exists a *competition* between color superconductivity and solidification, just like the case of laboratory low-temperature physics. One needs weak-interaction and low-mass in order to obtain a quantum fluid before solidification. This is why only helium, of all the elements, shows superfluid phenomenon though other noble elements have similar weak strength of interaction due to filled crusts of electrons. The strong color interaction (and the Coulomb interaction in the system with strangeness) may be responsible for a possible solidification of dense quark matter with low temperatures. Further experiments (in low-energy heavy ion colliders) may answer whether quark matter is in a state of solid or color-superconductivity. Can a solid neutron star be possible? The answer might be *no*, because at least the part of neutron matter with approximate nuclear saturation density should be in a fluid state. In this sense, only solid quark matter is possible, and a quark star is identified if one convinces that a pulsar is in a solid state.

We have assumed that the entire strain energy E_{strain} is relieved in a quake, which results in the reference oblateness of Eq. (7). However, it is possible that not entire, but most of, the energy E_{strain} is released in a real situation, and the actual reference points are near but larger than ε_i . Of course, there is a tendency of $\varepsilon \rightarrow \varepsilon_i$ after the i -th quake, but the effective shear modulus, μ_{eff} , of matter broken could be much smaller than that of perfect elastic solid, μ . The recovery timescale could be $\tau \sim 15$ days if μ_{eff} is order of 10^{15} erg/cm³.

A large Vela glitch on 2000 January 16.319 was noted [29], and *Chandra* observations were carried out ~ 3.5 and ~ 35 days after the glitch [30], but no temperature change expected in conventional models with released thermal energy of $\sim 10^{42}$ ergs is detected. This could be understood in this starquake model since (1) the thermal conductivity of

quark matter is much larger than that of hadron matter and (2) the thermal energy released, E_{therm} , to be much smaller than 10^{42} ergs is possible.

Acknowledgements

This work is supported by National Nature Sciences Foundation of China (10273001), and by the Special Funds for Major State Basic Research Projects of China (G2000077602). The valuable suggestions from an anonymous referee are sincerely acknowledged.

References

- [1] R.X. Xu, B. Zhang, G.J. Qiao, *Astropart. Phys.* 15 (2001) 101, and references therein.
- [2] R.X. Xu, in: X.D.Li, Z.R. Wang, V. Trimble (Eds.), *High Energy Processes, Phenomena in Astrophysics*, Proceedings of IAU Symposium No. 214, 2003, p. 191 (astro-ph/0211348).
- [3] R.X. Xu, in: K.S. Cheng, K.C. Leung, T.P. Li (Eds.), *Stellar astrophysics—a tribute to Helmut A. Abt*, Kluwer Academic Publishers, 2003, p. 73, <astro-ph/0211563>.
- [4] I.H. Stairs, A.G. Lyne, S.L. Shemar, *Nature* 406 (2000) 484.
- [5] T.V. Shabanova, A.G. Lyne, J.O. Urama, *Astrophys. J.* 552 (2001) 321.
- [6] J.E. Horvath, 2004, preprint <astro-ph/0404324>.
- [7] B. Link, *Phys. Rev. Lett.* 91 (2003) 101101.
- [8] B. Link, C. Cutler, *Mon. Not. R. Astron. Soc.* 336 (2002) 211.
- [9] Y. Levin, C. D’Angelo, 2004, preprint <astro-ph/0403014>.
- [10] R.X. Xu, *Astrophys. J.* 596 (2003) L59.
- [11] R.X. Xu, in: F. Camilo, B.M. Gaensler (Eds.), *Young neutron stars and their environments*, IAU Symp. 218, p. 299, <astro-ph/0310050>.
- [12] T.H.R. Skyrme, *Proc. R. Soc. London A* 260 (1962) 127.
- [13] R. Ouyed, M. Butler, *Astrophys. J.* 522 (1999) 453.
- [14] H.-J. Lee, B.-Y. Park, M. Rho, V. Vento, *Nucl. Phys. A* 726 (2003) 69.
- [15] M.A. Alpar, *Phys. Rev. Lett.* 58 (1987) 2152.
- [16] O.G. Benvenuto, J.E. Horvath, H. Vucetich, *Phys. Rev. Lett.* 64 (1990) 713.
- [17] H. Heiselberg, C.J. Pethick, E.F. Staubo, *Phys. Rev. Lett.* 70 (1993) 1355.
- [18] G. Baym, D. Pines, *Ann. Phys.* 66 (1971) 81.
- [19] F. Iachello, W.D. Langer, A. Lande, *Nucl. Phys. A* 219 (1974) 612.
- [20] V.R. Pandharipande, *Nucl. Phys. A* 248 (1975) 524.
- [21] M.A. Ruderman, *Nature* 223 (1969) 597.
- [22] C. Alcock, E. Farhi, A. Olinto, *Astrophys. J.* 310 (1986) 261.
- [23] M.S. Desai, H.J. Crawford, G.L. Shaw, *Phys. Rev. D* 47 (1993) 2063.
- [24] K. Fuchs, *Proc. R. Soc. London A* 153 (1936) 622.
- [25] T. Strohmayer, H.M. van Horn, S. Ogata, H. Iyetomi, S. Ichimaru, *Astrophys. J.* 375 (1991) 679.
- [26] M. Alford, K. Rajagopal, F. Wilczek, *Phys. Lett. B* 422 (1998) 247.
- [27] M. Alford, J. Bowers, K. Rajagopal, *Phys. Rev. D* 63 (2001) 074016.
- [28] M. Alford, K. Rajagopal, F. Wilczek, *Nucl. Phys. B* 537 (1999) 443.
- [29] R.G. Dodson, P.M. McCulloch, M.E. Costa, *IAU Circ.* 7347 (2000).
- [30] D. Helfand, E.V. Gotthelf, J.P. Halpern, *Astrophys. J.* 556 (2001) 380.